Zone Plate

Laser

8.5 mm or 16 mm TV lens

Zone Plate

Screen
pends to the portion of the vibration spiral extending from $B$ to $Z$. Thus an optical disturbance at $P$ is represented by the vector $BZ$. As the radius of the opaque disk increases gradually, the point $B$ moves along the spiral to $Z$, and the intensity decreases very slowly, without going through maxima and minima as it does in the case of a circular aperture. Also, no maxima or minima are observed as the point $P$ moves along the axis.

We can investigate qualitatively the distribution of intensity in a plane by considering the contributions of the various cone zones, as we have done in the case of a circular aperture. We again find that the diffraction pattern consists of a series of concentric rings alternately bright and dark. At the center, however, the intensity is always a maximum (Fig. 4-24).

Again we may discuss what happens when the radius $R$ of the disk is not very small compared with the distance $r_0$ of the point of observation. Suppose that the point of observation is slightly off the axis, or that the disk is perfectly circular in shape (so that several Fresnel zones are partially and partially obstructed). With an argument analogous to that developed in Section 4-5, we find that the vibration curve begins with a curvature, and spirals first outward and then inward again. If the lens of the opaque screen is sufficiently large, both the expanding and the shrinking portions of the spiral are tightly wound and the spiral ends at very nearly the same point where it begins (Fig. 4-16). The light intensity is as practically zero, in agreement with the prediction of geometrical optics.

4-5 Zone plates. A zone plate is a screen made of alternately transparent and opaque zones equal in dimensions to the Fresnel zones, and hence vering all even (or odd) zones. At the point of observation the exposed zones produce optical disturbances of the same phase. The resultant optical disturbance is therefore much greater than that observed in the absence of a screen, which, as we have seen, has an amplitude of half that produced by a central zone alone.

A zone plate is easily prepared by drawing a series of concentric circles of appropriate dimensions on white paper, painting alternate zones black, and then photographing the pattern thus obtained from a suitable distance (Fig. 4-26).

To compute the dimensions of the zones, we refer to Fig. 4-27, where $S$ presents the source, $P$ the point of observation, $O$ the plane of the zone plate (perpendicular to $SP$), and $O$ the intersection of $SP$ with $O$. Let $r_0$ and $r_1$ be the distances of $O$ from $S$ and $P$, respectively. We assume that these distances are large compared with the wavelength $\lambda$. As our xiliary surface we now take the plane $O$ of the zone plate. We must, of course, consider that this is not a wave surface, and so the virtual secondary rays are not in place. We denote by $R$ the distance of an arbitrary point $Q$ of this plane from $O$, and by $r$ and $r'$ the distances of $Q$ from $S$ and $P$, respectively. If we define

$$l = r - r_0, \quad l' = r' - r_0,$$

we have

$$(r_0 + l)^2 - r_0^2 = R^2,$$

$$(r_0' + l')^2 - r_0'^2 = R^2.$$

If we assume that $R$ is small compared with $r_0$ and $r_0'$, $l$ and $l'$ are also small compared with $r_0$ and $r_0'$ and the above equations, to a good approximation, yield

$$l = \frac{R^2}{2r_0}, \quad l' = \frac{R^2}{2r_0}.$$

Fig. 4-26. Zone plate.

The difference $l'$ between the distances of $Q$ and $O$ from $S$ introduces a phase delay $2\pi l/\lambda$ between the virtual secondary sources located at $Q$ and $O$, respectively. The difference $l'$ between the distances of $Q$ and $O$ from $P$ introduces an additional phase delay $2\pi l'/\lambda$ between the secondary waves traveling from $Q$ to $P$ and from $O$ to $P$, respectively. Hence the total phase difference between the two secondary waves at $P$ is $2\pi(l + l')/\lambda$.

The outer boundary of the first Fresnel zone is a circle of radius $R_1$ defined by the condition that the secondary waves originating from points of this circle arrive at $P'$ with a phase opposite to that of the secondary wave originating from the center $O$ of the circle. The radius $R_1$ is thus determined by the equation

$$l + l' = \frac{\lambda}{2},$$

which, together with (4-23), yields

$$R_1^2 \left( \frac{1}{2r_0} + \frac{1}{2r_0'} \right) = \frac{\lambda}{2}.$$

Fig. 4-27. Computation of the radii of the zones in a zone plate.
Similarly, the outer boundary of the second Fresnel zone has a radius \( R_2 \) determined by the equation

\[
R_1^2 \left( \frac{1}{2r_0} + \frac{1}{2r_0'} \right) = \lambda. \tag{4-25}
\]

In general, we obtain

\[
R_n^2 \left( \frac{1}{2r_0} + \frac{1}{2r_0'} \right) = n \frac{\lambda}{2}. \tag{4-26}
\]

Thus the boundaries of the successive zones are circles whose radii increase as the square roots of the integers, a result already obtained in Section 4-4 for the special case of a plane incident wave.

The actual widths of the individual zones depend on the positions of the source and the point of observation, i.e., on the values of \( r_0 \) and \( r_0' \). However, there are infinite pairs of values for \( r_0 \) and \( r_0' \) that correspond to zones of the same dimensions; indeed, they comprise all pairs satisfying the equation

\[
\frac{1}{r_0} + \frac{1}{r_0'} = \frac{\lambda}{R_1^2} \tag{4-27}
\]

(see Eq. 4-24). The values of \( r_0 \) and \( r_0' \) that satisfy (4-27) define two points that we may call conjugate with respect to a given zone plate. The zone plate concentrates a large fraction of the light coming from one of its conjugate points upon the second, much as a lens does. In fact, (4-27) becomes identical to the equation of a thin lens (Eq. 2-38) if we put \( r_0 = s, r_0' = s' \), and

\[
f = \frac{R_1^2}{\lambda}. \tag{4-28}
\]

4-9 The Cornu spiral. In the study of the Fresnel diffraction phenomena considered so far we have made use of the vibration curve described in Section 4-2, which refers to a subdivision of the wave surface into infinitesimal circular zones. For a certain class of diffraction phenomena it is more convenient, however, to subdivide the wave surface into infinitesimal rectilinear strips.

We consider a plane wave surface (Fig. 4-28) and a system of cartesian coordinates with the \( y \)- and \( z \)-axes in the plane of the wave, the origin at \( O \), and the \( x \)-axis through the point of observation \( P \). Let \( x_0 \) be the distance of \( P \) from the wave surface. We wish first to compute the disturbance produced at \( P \) by the secondary waves originating from points of an infinitesimal strip parallel to the \( z \)-axis, extending from \( y \) to \( y + \Delta y \). These waves