Experiment 01: Force and Motion

Purpose of the Experiment:
In this experiment you will study the motion of a cart on a track in response to an applied force. The experiment is intended to help you investigate the following ideas, many of which you may already know.

- Velocity is the rate of change of an object’s position with respect to time, and acceleration is the rate of change of an object’s velocity with respect to time.
- Compute the instantaneous and average velocity from (i) a position vs. time graph of an object’s motion and (ii) from a mathematical function describing the position of an object as a function of time.
- Understand that an object’s acceleration may be either constant or may change with time or position.
- Determine experimentally whether an object moving in one dimension has constant or variable acceleration.
- Describe mathematically the motion of objects with either constant or variable acceleration.
- Learn how to represent the motion of an object using words, graphs, numerical tables, and equations. Learn to move comfortably among the different ways to represent motion.

Setting Up the Apparatus:
You will use a PASCO cart rolling on a track. A motion sensor will let you measure the cart’s position as a function of time. The motion sensor works by sending ultrasonic pulses to the cart and detecting the return echoes.

First attach an adjustable end stop and a pulley to the end of the track as shown in the photo to the left. (The adjustable stop is to keep the cart from crashing into the pulley.) Adjust the leveling screw at the other end of the track to get the track as level as you can; test by checking that the car does not have a greater tendency to roll in one direction than another. One end of a string will attach to the cart and the other end will pass over the pulley and attach to a weight that provides the force to accelerate the cart.
The end of the cart with the two Velcro pads has a clear plastic plunger. You can release the plunger by pressing a button as shown at the right. Clip a $\frac{3}{4}$ inch binder clip to the plunger and push it back into the cart. Either a spring or a loop of string can be attached to the cart by passing it over one of the handles of the binder clip; to apply a balanced force, use the handle closest to the center of the cart. Tie a loop in one end of the string, and pass it over a handle of the clip, as shown in the photo below. Set the pulley height so the string from the cart to the pulley is horizontal.

To obtain a constant force to accelerate the cart, a 20 gm brass weight and its 5 gm plastic holder should be fastened to a loop in the other end of the string. Adjust the string length so that the falling weight hits the floor when the cart is about two inches from hitting the stop on the track.

To obtain a variable force to accelerate the cart, replace the brass weight and holder with a length of sash chain, as shown in the photo at the bottom of the first page.

The motion sensor should be placed at the other end of the track, against the stop that is permanently fastened to the track, as in the photo at the right. Make sure the slide switch on the top of the motion sensor is set to the narrow beam position. Connect the motion sensor to digital inputs 1 (yellow plug) and 2 of the SW750 interface to the computer. The apparatus is now ready to use.

The position of the cart will be measured using the motion sensor, and the data will be taken by the LabVIEW program ForceMotionLite.vi, which you can get from the course web page. The motion sensor measures the times for the echoes of ultrasonic pulses sent to the cart to return. The program calculates the distance to the cart (using a sound speed of 344 m/s). The program can plot the echo delay time (the “raw” data) or the distance to the cart. It can also calculate the velocity and acceleration of the cart and display them. The motion sensor works best if it is aimed slightly above the center of the cart rather than pointing directly at it. (That reduces the effect of sound waves that bounce off the track before hitting the cart; you may be having this problem if the position seems to jump as the cart rolls along the track.)
Using the LabVIEW Program:

Start the program (if it is not running) by clicking the white arrow below “Edit” in the top menu. When the program starts it will ask you for your 8.01 section and your table and group numbers. These are used to determine where to store any data that you might save.

When the program is running, you should get a window that looks like the one above.

The LabVIEW program ForceMotion.vi is controlled by a main pull-down menu above the graph to the left, under the label “Actions.” The items on the menu put the program into states to perform various actions; how to use it will be explained presently.

Many of the controls have small help dialogs that will tell you what they do if you put the cursor over them.

There are three tabs, labelled “Graph,” “Results” and “Chain”. These control what is displayed in the main part of the window.
Making a Measurement:

The first thing you will probably want to do is to make a measurement. To do that, hold the cart on the track about 15 cm from the motion sensor (the motion sensor may give silly results for objects closer than 15 cm), then choose Measure from the main menu and the RUN button will glow bright green.

As soon as you click the button (or type the Esc key) the program will start to measure the cart position, and will continue to do so until the cart has moved to be about 80 cm from the motion sensor. (If it does not stop automatically at this point, click the red STOP button or type the Esc key to stop it.)

The best way to make a measurement is to click the Graph tab, then click RUN (or type Esc), and then release the cart. You should see a graph of the raw data like the one below. On this graph, the \( y \) axis is the echo delay (in ms) of the ultrasonic pulse returned to the motion sensor and the \( x \) axis is the time elapsed since you clicked the START button. This what the program actually measures, but it is not what you want to know.

![Graph of raw data](image-url)
Plotting Data:

To the right above the graph there is a pull-down menu to control data plotting. The options are shown to the right. You can plot the cart position, velocity, or acceleration as a function of time. You can also return to the raw data plot if you wish.

After you have chosen what to plot from the menu, you must still ask the program to make the plot. That can be done either with the Plot Data option from the main menu or by clicking the Replot button just below the Plot Control menu. When you do that, the plot of your choice will appear on the graph.

If you click the Results tab you will see a table that contains the numbers that are plotted on the graph.

Manipulating the Plot:

The graph control palette above the graph (see picture at right) gives you some control over the graph. You will find the left and center buttons on the palette to be the most useful.

If you click the center (zoom) button to select it, a small window opens that will let you expand the scale of the plot by dragging the pointer over the region you want to expand. You can choose to expand either a range of the $x$-axis, a range of the $y$-axis, or you can drag to select a rectangular range in $x$ and $y$ and expand it to fill the graph window—depending on the option you click on in the small window. There is also an option to return the graph window to show all of the data. As soon as you have some data plotted on the graph, you should experiment with this control.

If you select the right button (the hand) you can drag the visible plot window (whose scale you chose with the zoom button) to wherever you want.

The graph has two cursors: the red and blue cross hairs you can see. To use the cursors, the left (cursor) button on the graph control palette must be selected. Then you can position the cursors by dragging them; they will jump to the data points. You can read the $x$ and $y$ coordinates of the cursor positions at the top of the graph.

Performing the Experiment:

Once you have played with the apparatus enough that you know how to make a measurement and plot the results, you are ready to proceed with today’s experiment. The goal is not to carry out a careful quantitative analysis of your measurements, but to see how much you can learn by examining graphs of your results. The next page has a step-by-step list of what to do. Refer back to pages 4 and 5 if you need to remind yourself how to operate the apparatus.

Pages 8–10 have a more detailed discussion of the quantitative data analysis capabilities of this program, for the benefit of the instructors. You are not responsible for any of this material, although you are free to read it.
What You Should Do:

Measure the motion of the cart in two cases:

1. When a fixed weight, say 25 gm, is attached to the string and accelerates the cart.
2. When a sash chain is attached to the string and accelerates the cart.

Before you close the program, answer the following questions.

1. In case 1, is the acceleration constant for most of the cart’s motion, or not? Explain how you can determine this using a graph of velocity vs. time.

2. In case 2, is the acceleration constant for most of the cart’s motion, or not? Explain how you can determine this using a graph of velocity vs. time.

3. In which of the two cases, if at all, can you describe the position of the cart as $x = x_0 + v_0t + \frac{1}{2}at^2$? Explain your answer for each case.

4. The empty cart has a mass of 250 gm. For the case of constant acceleration, what was the force (numerical answers) causing the acceleration? How much mass was being accelerated? Was the acceleration you determined consistent with Newton’s 2nd law, $F = ma$?

5. Suppose that your friend (who is a much better experimenter than most of us are) obtained a velocity vs. time graph that looks like this.

On the empty graphs on the next page sketch the position and acceleration vs. time graphs that correspond to the above velocity vs. time plot. Make sure the graphs you sketch are to scale and label the vertical axis.
Appendix: for Instructors Only

Data Massaging:
The program measures only the distance from the cart to the motion sensor, yet we would like to see graphs of the velocity and acceleration of the cart. A simple point by point numerical differentiation of the position curve introduces a lot of uncertainty and noise. Doing it twice to get the acceleration would give a plot that looks like a random distribution of data points. One could fit the position vs. time curve to an expression like $x_0 + v_0t + \frac{1}{2}at^2$, but this would give only the average velocity and acceleration and certainly fail if the acceleration were not constant.

A method to obtain derivatives of data that have some noise, and to reduce some of the noise itself, was introduced by two scientists at the Perkin Elmer Company in the 1960s [A. Savitzky and M. J. E. Golay, Analytical Chemistry 36, 1627-1639 (1964)]. The method is to carry out a local least squares fit of a polynomial to just a few data points near the time you are interested in, and then find the local derivatives from the fit coefficients. This LabVIEW program does that in order to make the velocity and acceleration plots. Even with this approach, the acceleration graph is rather noisy.

Using the Savitzky-Golay analysis, one can make two choices: (i) how many data points to include in the local fit, and (ii) what degree of polynomial to use in the fit. The defaults used in the program are 7 points (i.e., the point where you want the derivative and 3 points before and after it) and a quadratic polynomial—which can give first and second derivatives. The pull-down menus at the top of the Results tab (figure at left) offer other choices for anyone who is inclined to experiment.

The coefficient of the constant term in the local fit represents a local average and provides smoothing of the original data. Normally the “raw” position data are plotted on the graph and used in a fit, but a pull-down menu allows plotting the smoothed data instead.

You may want to experiment with this data manipulation; if you make any changes in the Savitzky-Golay pull-down menus, or even just the standard deviation for the position measurements, choose the bottom item on the main menu (Recalculate V, A) in order to apply them. Incidentally, whether you fit the raw or smoothed position data, you should get the same results.

Quantitative Data Analysis:
We do not ask the students to do this, but the program can carry out several least squares fits if you want to try quantitative data analysis. The program can carry out a least squares fit by three possible functions: $y = A$ (Constant), $y = A + Bx$ (Linear), or $y = A + Bx + Cx^2$ (Quadratic), where the $x$ variable is always the time, $t$. You choose the fit you want from the Fit Function? pull-down menu on the Results tab. You must also choose the range of data that will be included in the fit. Only the data points between the red and blue cursors on the graph will be fit, giving you a lot of flexibility. When you do the fit (Fit Plotted Data from the main menu) the data that are plotted on the graph will be fit.
**Acceleration By a Variable Force:**
A variable force is obtained by using the weight of a falling chain to accelerate the cart. It is clear from the graphs that the acceleration of the cart is not constant. The problem can be solved considering conservation of energy, or by using Newton’s law $F = Ma$. Here I use $F = Ma$ to calculate the motion of the cart. Suppose the chain has a density $\rho$ and at the start of the measurement the weight of a length $L$ of chain provides the force to accelerate the cart, whose mass is $M$. After the cart has moved a distance $x$ the length of chain whose weight accelerates the cart is $L - x$. The force doing the acceleration is therefore $g \rho (L - x)$, and the mass being accelerated is $M + \rho (L - x)$. This gives an equation of motion:

$$[M + \rho (L - x)] \frac{d^2x}{dt^2} = \rho g (L - x)$$

This equation should be solved for the range $0 \leq x \leq L$; if $x > L$, the cart will just move at a constant velocity. The equation will look simpler if we change variables, replacing $M + \rho (L - x)$ by $y$. Then $y = M + \rho L$ when $x = 0$ and $dy = -\rho \, dx$. When this substitution is made, the equation becomes

$$\frac{d^2y}{dt^2} = \rho g \frac{M - y}{y}$$

This is not an easy equation to solve for $y(t)$, but we can make some progress by introducing a new variable $\eta = dy/dt$. Then we have (using the chain rule for derivatives)

$$\frac{d^2y}{dt^2} = \frac{d\eta}{dt} = \frac{dy}{dt} \frac{d\eta}{dy} = \eta \frac{d\eta}{dy} = \rho g \frac{M - y}{y}$$

This has given us a first order differential equation relating $\eta$ to $y$; this equation is

$$\eta \frac{d\eta}{dy} = \rho g \frac{M - y}{y} \, dy = \rho g \left( \frac{M}{y} - 1 \right) \, dy$$

This equation can be integrated by anyone with a rudimentary knowledge of differential calculus. The result is

$$\frac{1}{2} \eta^2 = \rho g (M \ln y - y) + C.$$ 

The constant of integration $C$ can be found by recognizing that the cart starts out from rest ($\eta = 0$ when $x = 0$ or $y = M + \rho L$). This requires

$$C = \rho g [M + \rho L - M \ln(M + \rho L)]$$

and gives the result

$$\eta^2 = 2 \rho g [M + \rho L - y + M \ln y - M \ln(M + \rho L)]$$
To return to the original variable $x$, replace $y$ by $M + \rho(L - x)$ and get

$$\eta^2 = \rho^2 \left( \frac{dx}{dt} \right)^2 = \rho^2 v^2 = 2\rho g \left[ \rho x + M \ln \left( 1 - \frac{\rho x}{M + \rho L} \right) \right].$$

The final result gives a relation between the speed of the cart $v$ and the distance $x$ it has moved down the track

$$v = \sqrt{2g \left[ x + \frac{M}{\rho} \ln \left( 1 - \frac{x}{L + M/\rho} \right) \right]}.$$

The maximum speed, when $x = L$, is

$$v_{\text{max}} = \sqrt{2gL \left[ 1 + \frac{M}{\rho L} \ln \left( \frac{M}{M + \rho L} \right) \right]}.$$

The program cannot fit this expression to the data, but it can calculate the expression above for $V(x)$ and plot it on the same graph as the data. If you click the Chain tab, there are fields where you can type in the mass of the cart and the mass per unit length of the chain. Then if you choose plot V vs. X from the plot menu, replot the data, and then select Fit Plotted Data from the main pull-down menu the expression will be evaluated and plotted on the graph. You can choose different values for chain mass per unit length (the cart mass should be well known) and see how closely the expression comes to the data.