In-Class Problems: Potential Energy Differences and Conservation of Mechanical Energy

Section_______ Table and Group Number ______________________

Names __________________________
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Hand in one solution per group.

We would like each group to apply the problem solving strategy with the four stages (see below) to answer the following two problems.

1. Understand – get a conceptual grasp of the problem
2. Devise a Plan - set up a procedure to obtain the desired solution
3. Carry our your plan – solve the problem!
4. Look Back – check your solution and method of solution

In-Class Problem Potential Energy Differences

For each of the three cases below:

1) Briefly describe the system and the force of interaction. Introduce coordinate system and any variables as necessary.

2) Calculate the integrals below in order to determine the potential energy difference for each force of interaction.

3) For each of the motions described by the initial and final states below: choose a zero point for the potential energy and briefly describe why you made that choice, predict the sign and potential energy difference and briefly describe your reasoning, and use your results from step 2) to calculate the potential energy difference.

\[
U(y_f) - U(y_0) = \int_{y_0}^{y_f} \vec{F}_{\text{gravity}} \cdot d\vec{r}, \text{ where } \vec{F}_{\text{gravity}} = -mg \hat{j}, \text{ and } d\vec{r} = dy \hat{j}.
\]
Initial state: a body is just released with a non-zero speed and non-zero angle with respect to the horizontal at the surface of the earth.

Final state: the body is at the top of its trajectory.

$$b) \ U(x_f) - U(x_0) = -\int_{x=x_0}^{x=x_f} \vec{F}_{\text{spring}} \cdot d\vec{r}, \ \text{where} \ \vec{F}_{\text{spring}} = -kx \ \hat{i}, \ \text{and} \ \ d\vec{r} = dx \ \hat{i}. $$

Initial state: a compressed spring is just allowed to expand.

Final state: the spring just reaches its unstretched length.

$$c) \ U(r_f) - U(r_0) = -\int_{r=r_0}^{r=r_f} \vec{F}_{\text{gravity}} \cdot d\vec{r}, \ \text{where} \ \vec{F}_{\text{gravity}} = -G \frac{mm_{\text{star}}}{r^2} \ \hat{r}, \ \text{and} \ d\vec{r} = dr \ \hat{r}. $$

Initial state: a body is moving towards a star but is very far away.

Final state: the body just reaches its distance of closest approach the star.
In-Class Problem Conservation of Mechanical Energy: escape velocity

The asteroid Toro, discovered in 1964, has a radius of about $R = 5.0 \text{km}$ and a mass of about $m_r = 2.0 \times 10^{15} \text{kg}$. Let’s assume that Toro is a perfectly uniform sphere. What is the escape velocity for an object of mass $m$ on the surface of Toro? Could a person reach this speed (on Earth) by running?