Note: Experiment 02 consists of two parts, each carried out on the same apparatus. The first part concerns static equilibrium, and the second is a study of friction. Different LabVIEW programs are used for each experiment, and there is separate documentation for each. The two different documents are each part of this file.
Purpose of the Experiment:
In this experiment you will measure and analyze the forces when bodies are in static equilibrium:

- A weight is suspended from a string that spans the horizontal distance between two horizontal support points. You will measure the tension in the string and see how it depends upon the amount of “slack” in the string. You will plot your results and compare them to the expected mathematical expression.

Suspending a Weight:

In this experiment, you will hang a 100 gm brass weight (the brass + holder mass is 105 gm, so the weight is 1.03 N) from the center of a string between supports as shown in the photo above.
As you can figure out from the free body diagram to the right, when the weight is hung from the middle of the string the two tensions are equal and given by

\[ T = \frac{Mg}{2 \sin \theta} \, . \]

The LabVIEW program StaticEqm can measure the tension in the string and calculate \( \theta \) from length measurements you will provide. Then you will use the program to plot tension vs. \( \theta \) and carry out a fit to see if you can verify the equation above.

Here is a technique that I found worked fairly well. Position the sliding T-square far enough to the right of center so that it does not interfere with the brass weight. Align an edge of the ruler with the center of a column of holes. As the holes in the board are on 1/2 in centers and the ruler is 1 in wide, the horizontal distance from the string support point (the pulley) to the ruler edges will be a multiple of 1/2 in. I placed the left edge of the ruler 8 in (or 203 mm) from the center of the pulley (you should measure this distance as carefully as you can), with the mm scale at the left edge. Keep this same horizontal distance for all of your measurements.
Running the Program *StaticEqm*:
To begin, plug the force sensor into the SW750 analog interface A, make sure the SW750 is connected to the computer by a USB cable, turn it on, and start the program. When the program starts, a dialog window will open for you to enter your section, table, and group. This information is stored in any files you save and also determines where the files will be stored on the computer.

If you want your measurements to be saved as a run number other than the one you see in the box, type that number into the the box. Most of the other fields you can see if you click the Run and Fit Parameters tab will be filled in as the experiment runs. There are three optional text fields where you can type anything you want. They will be saved in the file with your data.

Below the 8.01 logo there is another pull-down menu to control what the program does. If you click on it you will see the possibilities. You are ready to begin measurements, so choose the “Measure” option. When you do that, the “START” button will glow brighter green, indicating the program is ready to measure the tension. If you click the button it will change to red and say “STOP”. The program is now measuring the tension, but not recording any data. You can push on the force sensor’s hook and pull on the string and see the meter change. When you click the red STOP button, the measuring will stop, but do not do that until you have measured all of the data points you want.
Make sure there is no tension on the string and set the zero on the PASCO force sensor by pressing the “tare” button. Then, with no weight hanging from the string, pull it tight so the meter reads about 6 N and clamp it with the wingnut. The string should now mark a horizontal line. Slide the ruler up or down along the T-square so that the string passes in front of one of the cm markings; this will make it easier to measure the vertical drop of the string along the ruler edge when the weight is hanging on it.

Hang the 1.03 N weight from the center of the string. The weight should pull the center of the string about 15 mm below the horizontal position. You should measure the horizontal run from the pulley and the vertical drop to the point where the string crosses the edge of the ruler. These two lengths will be used to calculate the angle \( \theta \). When you have measured the two lengths, you are ready to have the computer measure the tension in the string and record a data point. If necessary, move the ruler so it does not interfere with the string and give a false tension reading. To record a measurement, click the “MEASURE” button. The program will measure the tension in the string, the button will change to yellow, and a dialog window will open asking you to type in the horizontal run and the vertical drop.

![String Position?](image)

Type in the values and click “OK”. The MEASURE button will change back to green, indicating it is ready for the next measurement. If you click the appropriate tabs, you will see that a point has been plotted on the graph and also entered into the data table.

To complete the remaining measurements, loosen the string so that the weight is lower by 10 mm to 15 mm each time, make sure the weight is hanging from the center of the string, and click the MEASURE button. Repeat this until the weight is as low as it can go without touching the table.

If you find you make a mistake typing numbers into the dialog window, just repeat the measurement by clicking the MEASURE button again. You will be able to remove the incorrect entries from the table later. If you use the same horizontal “run” for each measurement, the dialog box remembers it and you will not have to type it each time. When you have taken all the measurements you want, you are ready to see if your measurements agree with the theoretical prediction.

If you made any typing mistakes, remove the bad entries from the table before you click the STOP button. Here’s how to do that. On the graph I could see a “bad” point at \( \theta \approx 0.05 \) and \( T \approx 1.1 \) because I had typed the vertical drop in cm instead of mm. The first step is to find the bad point in the table.
Then select the bad entry in the table and right-click on it. That will open a menu like the one to the right. Choose “Delete Row”.

Be careful not to choose any of the other options. Some of them will destroy all of your data.

After you have deleted any bad points from the table, you should click the red STOP button so the program is no longer in the Measure mode.

The next step is to see how well the theoretical expression $T = A/\sin \theta$ describes your data.
Fitting Data:
The way to see how well a mathematical expression describes your measurements is to “fit” it to your data. That means that the adjustable parameters in the expression should be chosen to provide the best possible description of or fit to the data. We will do a lot of that when analyzing experimental results in 8.01.

In this experiment, the theoretical expression is

\[ T = A/\sin\theta + B, \]

which includes an optional background term \( B \). One adjustable parameter is the amplitude \( A \), and a second is \( B \), if chosen. If you click the Run and Fit Parameters tab, you will see a small pull-down menu labeled Background that is used to include the background term. Choose “Yes” and then “Fit Data” from the main pull-down menu. This will carry out the fit and plot the best fit function as a green line on the graph. You can look at it to see how well it seems to describe the data.

The parameters \( A \) and \( B \) that give the best fit are the ones that minimize the total squared error \( \sum (T(\theta_i) - A/\sin(\theta_i) - B)^2 \).

There is a useful graph control palette at the top of the graph. The center button is a zoom control that allows you to expand regions of the plot to fill the window.

The left button controls a cursor on the graph; the graph in this program does not have a cursor, but LabVIEW programs future experiments will use one or two cursors. The “hand” control allows you to drag the visible window to different regions on the expanded graph. Experiment with the controls to see what they do, as they will be helpful when working with graphs.

To get a more quantitative indication of the agreement between the measurements and the theoretical expression, click the “Run and Fit Parameters” tab again.

The data that were fit will be there in the table, and some of the other boxes will have numbers in them; these are the results of the fit. One of the important ones is labeled “Root MSE”. This is the square root of the mean squared error; the latter is the total squared error divided by the number of fitted data points. Thus the Root MSE is the magnitude of the average difference between the mathematical fitting function and the data points, and is one measure of how good the fit is. In my experiment it was about 0.026 N. PASCO claims the force sensor has a resolution of ±0.03 N and output noise of ±0.012 N. These would combine to give a measurement error of about \( \pm\sqrt{0.03^2 + 0.012^2} \approx \pm0.032 \) N. This is the standard deviation to be expected for each measurement of the tension, and is usually represented by the letter \( \sigma \). Thus the theoretical expression fits within the measurement accuracy the force sensor is capable of.

I found the value of the amplitude coefficient \( A = 0.513 \) N. An important question is how accurately was \( A \) determined from the fit? The program gave a standard deviation of ±0.002 N. Where did this come from? There is a commonly accepted procedure. The standard deviation is the change that must be made in the parameter \( A \) from its best fit value in order to increase the Root MSE by \( \sigma \).
If there is more than one parameter, the others are changed to keep the total squared error as small as possible while the parameter of interest is varied. The StaticEqm program allows you to fit the data to an expression with a background term

\[ T = \frac{A}{\sin \theta} + B, \]

and that fit gave me \( B = -0.037 \pm 0.012 \text{N} \). There is no theoretical reason to expect a background, unless you forget to tare the force sensor, so I did the fit again with no background. With my data, when I did the fit again without the background there was no visible difference on the graph, and only a slight difference in the numbers on the Run and Fit Parameters page. You should also try a fit with no background.

With an adjustable background, the Root MSE was slightly smaller. A smaller total squared error should be expected when there are more adjustable parameters. Therefore the custom is not to consider simply the mean squared error but to take the effect of more parameters into account by dividing the total squared error by one less than the number of points in the fit less the number of adjustable parameters (i.e., \( N_{\text{points}} - N_{\text{params}} - 1 \)). This result is then compared to (i.e., divided by) the square of the expected standard deviation \( \sigma \) of the tension data points. The quantity calculated this way is called “chi-squared” or \( \chi^2 \). Mathematically, this is

\[
\chi^2 = \sum_i \frac{[T(\theta_i) - A/\sin(\theta_i) - B]^2}{(N_{\text{data}} - N_{\text{params}} - 1)\sigma^2}.
\]

If \( \sigma \) is correctly known, an acceptable fit should have \( \chi^2 \approx 1 \). The value of \( \sigma \) can be typed into the field Y StdDev on the tab, so you repeat the fit with different values of \( \sigma \) and see how they change \( \chi^2 \). The program also gives you the option to fit only the first \( N \) data points, where \( 4 \leq N_{\text{data}} \leq 10 \); you might want to experiment with this option. Using \( N_{\text{data}} = 13 \) (all of my data points) and \( \sigma = 0.03 \), I found \( \chi^2 = 0.9 \) with the background term, and \( \chi^2 = 1.6 \) without it. (Fits are not considered statistically different unless one has a \( \chi^2 \) that is at least \( 1/(N_{\text{points}} - N_{\text{params}} - 1) \) greater.) I found that the standard deviation of the background term is about \( 1/3 \) of its value; that’s all a pretty clear indication that the background term is marginal and should probably be taken to be zero.

The program also calculated something labeled the “Covariance Matrix.” If you get a calculator, you can see that the diagonal elements of this matrix are the squares of the standard deviations of the parameters \( A \) and \( B \). (The square of the standard deviation is called the variance.)

If there is more than one adjustable parameter, you can imagine that a change in one parameter might be compensated for by a change in another. If that happens, the parameters are said to be correlated. The off-diagonal terms in the covariance matrix are a measure of this correlation. Sometimes when you determine parameters by fitting data you would like to know the uncertainty in a quantity that is a function of the fit parameters; that uncertainty can be calculated using the covariance matrix and we will do that in some future experiments.

This, and the discussion for the force law experiment, has been a short introduction to some of the ideas used in data analysis. We will meet them again in future experiments, and I guarantee that—no matter what major you choose at MIT—you will need to use them in the future.
For this experiment, if your results are like mine, the conclusion is that the tension in the string is given by the theoretical expression $T = A/\sin \theta$ to within the accuracy of the measurement.

**Saving Your Results:**
One of the menu options is to save your measurements. The file can be read by the *StaticEqm* program and by spreadsheets like Excel if you would like to analyze the data later or print them. Before saving the file, click the Run and Fit Parameters tab and type any pertinent remarks into the comment lines. (The Run Identifier field will be filled in by the computer.) Then choose “Save Data” from the menu. The computer will suggest a folder to save the file in and a file name; you are free to make different choices, but the suggested one will put the data in a place where it will be accessible to all of the members of your group, not just whoever happened to log in at the start of the class.

The data are stored in an ASCII text file that you can examine or edit with a text editor such as *Notepad*.

**Retrieving Saved Results:**
Choose “Read Data File” from the menu and select the file you want to read. If it contains the kind of data that *StaticEqm* expects, they will be plotted on the graph and entered in the table. You may fit them in the same way as if you had just measured them.

You can get copies of the LabVIEW program *StaticEqm* that will run on your own computer (Linux, Macintosh, or Windows) and it has been installed on Athena so that you can run it that way, too. See the link about LabVIEW on the course home page for more information.

**A Reference**
My favorite book on data analysis is *Data Reduction and Error Analysis for the Physical Sciences*, by Philip R. Bevington. Bevington died in 1980, but the book has been revised and updated by his co-author Keith Robinson. It is not necessary for 8.01, but sooner or later in your career, you will want to own this book or its equivalent.
Purpose of the Experiment:
In this experiment you will study the friction between a string and a plastic cylinder. You should observe:

- The ideal case where there is a coefficient of static friction that is obviously greater than the coefficient of sliding friction may not apply in all situations.
- The friction force between the string and cylinder increases exponentially with the distance the string is wrapped around the cylinder.
- You will measure an effective coefficient of friction by fitting an exponential function to your data.

Setting Up the Apparatus:

Use the same apparatus as for the static equilibrium experiment, but remove the sliding T-square and attach the plastic cylinder near the top center of the board. The cylinder has a handle so you can turn it (although I preferred to grab the end of the cylinder with my fingers) and two regions for wrapping the string. One has a diameter of 0.5 in and the other of 1.0 in. You may use either one, but discuss with the other groups at your table to make sure that both diameters are being used at your table.

The diameter you are using should be placed closest to the mounting board; note that the two diameters use different mounting holes. Choose the mounting hole so that a string passing over the pulley to the force sensor and around the plastic cylinder will be horizontal.
Here are photos showing more details of how the cylinder may be used.

One end of the string passes over or wraps around the cylinder and is attached to a brass weight by a plastic holder. Try to avoid wrapping the string over itself as it goes around the cylinder. The other end passes over a bearing as a low-friction pulley to transmit the tension force to the force sensor. It is very important for the string to pass over this pulley for all your measurements so that the force will be measured correctly.

Choose a brass weight of mass 100 gm; then the weight of the brass plus holder (105 gm mass) will be 1.03 N.
Measuring the Friction:

To begin, plug the force sensor into input A of the SW750 interface and start up the LabVIEW program Friction.exe. The program will ask for your section, table, and group so that it knows where to store your data. The program operation is controlled by the pull-down menu shown in the figure below.

Select the Measure option to measure the tension in the string. The RUN button will turn bright green to indicate it is ready. To make a measurement, wind the string $n + \frac{1}{4}$ turns around the cylinder and hang the weight from it, as in the photos on the second page. You should make measurements with $n = 0, 1, 2, 3, 4, \text{ and } 5$. Before each measurement, hold the weight so there is no tension in the string and tare the force sensor. Make sure the string passes over the pulley, the string does not overlap itself as it wraps around the cylinder, and let the weight hang. Grasp the cylinder and twist it counter-clockwise. As you increase your twisting force you should reach a point where the string starts to slip. Practice this a few times with $2 + \frac{1}{4}$ turns so you get the feel of it and are able to smoothly increase the torque to the point where slipping starts and are able to keep it slipping for a second or two.

The Sample Rate should be set to 20 Hz and the Window to 10 s. When you click the RUN button it will change to red and say STOP; the program will measure the tension in the string to the force sensor 20 times a second and plot it on the graph. It will keep only the most recent 10 seconds of measurements in memory and on the graph. When you click the red STOP button, it will stop taking measurements. The program will then open a dialog window and ask you how many turns of string were wrapped around the cylinder.

Choose from the pull-down menu. You should use the 0 Turns option to measure the weight (1.03 N) of the brass weight when the string is draped gently over the cylinder.
If you were twisting the cylinder, you should obtain a graph that looks something like this one.

The *Friction* program calls this an “Initial Scan” graph. The data from the initial scans are not entered into a table on the Fit Results tab (they only appear on the graph), but they may be saved to a data file on the computer.

The graph has a cursor on it. To use the cursor, make sure the left button on the graph control palette is selected.

You can position the cursor by dragging it or use the arrow keys at the top of the graph or the keyboard right/left arrow keys. It will hop from one data point to the next as it moves. On the graph, you want to choose the data point you think represents the tension in the string when it just starts to slip as you twist the cylinder. It is probably the point with the greatest tension.

If you want to save the data on the graph, this would be a good time to do it; if you do save it (recommended), enter the cylinder diameter—either 1.0 or 0.5 inches—and weight into comment fields on the Fit Results tab before you do so.

The next step is to have the computer record the tension at which the string began to slip. To do that, position the cursor on the point you choose and click the bright green SELECT POINT button. After you do, an entry will appear in the Theta/Tension table under the Fit Results tab.

Now you can repeat the process for different numbers of turns of string wrapped around the cylinder: select Measure from the main pull-down menu, make your measurement, and choose the data point you think represents the tension where the string started to slip. Do this, increasing the wrap by one turn each time up to 5 1/2 turns. You should end up with a Theta/Tension table that has an entry for each measurement you made.

The program calls this “Result Summary” data.
At this point, fit the data in the table to the theoretical expression $T = Ae^{\mu \theta}$. Choose the Fit Function “Exponential”. The numerical fit results will appear on the Fit Results tab, and the data points from the table as well as the fit function will be plotted on the graph.

There is no reason to have a background term $B$ if the force sensor was properly tared, but choosing the Fit Function “Exponential + BG” will include one. Try that fit as well and compare with what you found when fitting with no background.

You should save your Result Summary data, too.

Think about the following questions:

- Was the fit significantly better when a background term was included?
- How does the standard deviation of $B$ compare with $B$ itself?
- How do the values of the friction coefficient $\mu$ differ from the two fits, and how does that difference compare with the standard deviations calculated in the fits?

The data in the Theta/Tension table can be saved to a file as “Result Summary” data. You should compare results with other students who used a different diameter cylinder to see if they obtained exponential behavior with the same $\mu$. 

Experiment 02B 14 September 14, 2005