Experiment 03: Projectile Motion

Purpose of the Experiment:
In this experiment you will study projectile motion and analyze your results to determine the value of $g$.

- You should be able to verify the kinematic equation describing projectile motion by assuming that it is correct and seeing that it predicts the correct value for $g$ and for the range of the projectile.
- You will see how the experimental errors in the quantities you measure combine to produce errors in $g$ and the projectile range. This process is called error propagation.

Setting Up the Apparatus:
The apparatus has a hollow plastic tube that can be used as a “gun” to launch the projectile, a 12 mm diameter steel ball bearing. The angle of the tube may be changed in 15° increments (determined by threaded inserts in the backboard) and the apparatus may be oriented so the initial vertical component of the projectile’s velocity is either upward or downward. If upward, a spring gun imparts the initial velocity; if downward, you should allow the initial velocity to be determined by gravity—otherwise the initial velocity will be too great for accurate measurements. The photo below shows how to position the apparatus for a downward launch.
For an upward launch, the apparatus is positioned as in the photo to the left. In either case, the apparatus should be clamped to the table.

At the output of the launching tube, the ball interrupts the beam of light from an LED to a phototransistor to produce a signal as it leaves the tube. Terminals on the apparatus enable connection to a power supply for the LED and to the output of the phototransistor. The latter should be connected to input channel A on the SW750 interface.

Place the clipboard so the ball will land on it. A piezoelectric buzzer is attached to the clipboard; it acts as a microphone so you can measure the time the ball hits the clipboard. Connect the terminals of the piezoelectric buzzer to input B of the SW750.

In this experiment, you should measure at least one upward and one downward launch. Measurements are a bit easier for the downward launch and you may want to do it first. Try some practice launches to make sure you have the clipboard in the right place.

At the output end of the launch tube is a piece of white plastic that contains the LED and phototransistor; there is a small hole about 1 mm diameter where the light beam crosses the tube (see the pointing finger in the photo at the right). You should measure as accurately as you can the vertical distance from this hole to the clipboard and subtract the radius (0.006 m) of the ball to get the height $h$ from the launch point to the impact point of the projectile. Later you will also measure the horizontal distance from the light beam to the impact point of the ball, which will be the range $r$ of the projectile.

Place a piece of white paper on the clipboard and cover it with a piece of carbon paper with the carbon side down. The impact will make a mark on the white paper that allows you to determine where the ball landed.
Making the Measurements
Start the LabVIEW program *ProjectileMotion.exe*. The program is controlled by the main pull-down menu at the left above the graph.

The program will measure the input voltages on channels A and B of the SW750. These voltages are normally close to zero, but the phototransistor voltage (channel A) rises to about 6 V when the light beam is blocked by the ball bearing. The program does not begin to record data until this voltage exceeds a trigger value. Use the slider to the left of the graph to set this value; the default 1.0 V should work well. There is also a vertical indicator of the phototransistor voltage that is active when the RUN button has changed to red and reads STOP; it is there for diagnostic purposes and you shouldn’t need it.

Once the trigger condition has occurred, the program will record the voltages for a time determined by the window setting. The default value of 0.8 s should be OK. The window setting should be long enough that the signal from the ball bearing hitting the clipboard is recorded. The voltages will be measured every 200 µs.
To make a measurement, choose the Measure option from the menu. The RUN button will change to bright green, indicating the program is ready. When you are ready to launch the projectile, click the RUN button (or the Esc key) and then launch the ball bearing. After the window time has passed, the RUN/STOP button should change from red back to green and you should have a graph like the one on the previous page. The phototransistor voltage is blue and the microphone output is green. The next step is to use the cursor to determine three important times during the projectile motion.

Make sure the left (cursor) button on the graph control palette is selected, and move the cursor (red) on top of the phototransistor voltage pulse at the left edge of the graph.

Use the zoom control (select the center button) to expand about the first 20 ms of the graph to the full horizontal scale. This is most easily done by clicking on a point just to the right of the phototransistor output and dragging just off the left edge of the graph. Then you will see the phototransistor output pulse clearly.

The leading edge of the ball is half way across the light beam when the voltage pulse has risen to half its maximum value, and the trailing edge of the ball is half way across the light beam when the pulse has fallen to half its maximum value. Thus the launch speed of the ball is 

\[ v_0 = \frac{D}{\tau} \]

where \( D \) is the ball diameter and \( \tau \) is the full width at half maximum of the phototransistor output pulse.

Select the cursor button on the palette again. Position the cursor on the data point that is closest to half way up the rising edge of the pulse and click the green RISE button in the box labeled “Select Time of...”; that tells the computer the time that the leading edge of the ball was half way across the light beam. Move the cursor to the falling edge of the pulse and repeat the process, but click the green FALL button. (The cursor stops only on actual data points, so you may not be able to position it exactly half way up or down the pulse.)
If you chose a point slightly more than half way up for the rise, then pick one slightly more than half way down for the fall, or vice versa. That should reduce the error in determining $v_0$. The computer subtracts these two times to get $\tau$ and averages them to get the launch time for the projectile.

Next, determine the impact time of the ball on the clipboard. Use the zoom control to set the graph window to show the entire measurement and position the cursor close to the green voltage pulse from the microphone. Then expand that region to fill most of the graph window.

Place the cursor at the start of the pulse and click the green IMPACT button. Now you are ready to complete the data analysis. Click the Results tab.
The Rise, Fall, and Impact times should be displayed to the right of the bottom visible row of the table. Type the launch angle (in degrees, positive for an upward launch and negative for a downward one) and the height you measured into the Input Parameters table. Choose Analyze Data from the pull-down menu, and the rest of the fields will be calculated by the program. The three times you selected with the cursor, the time of flight, and the initial speed \(v_0\) are all shown near the bottom of the tab. They will also be entered, along with the height and angle \(\theta\), into two of the comment text fields.

The value of \(g\) calculated by the program, the projectile range \(r\), and their standard deviations are given in the Output Parameters box.

You can also change the values for the standard deviations of \(h\) and \(\theta\) and repeat the calculation, but the defaults (\(\sigma_h = 0.01\) m and \(\sigma_\theta = 2^\circ\)) should be OK, as are the defaults for \(D\) and \(\sigma_D\). Be sure to save your data; this would be a good time to do it.

You should measure the range and compare it with the calculated range. Go to the “Data Input Page” link (on the Experiments page for the course) and enter your value for \(g\). Then your instructor will be able to plot a histogram showing all the results in your section.

<table>
<thead>
<tr>
<th>Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Input Page (Students)</td>
</tr>
</tbody>
</table>

**Data Analysis**

Here is how the *ProjectileMotion* program analyzed your measurements to find \(g\), \(r\), and their standard deviations. The time of flight is \(T\) and the full width at half maximum (FWHM) of the pulse is \(\tau\); \(\tau\) is the time for the ball to travel its own diameter \(D\). (Thus \(v_0 = D/\tau\).) Then the range \(r\) of the ball is

\[
r = v_0 T \cos \theta = \frac{DT \cos \theta}{\tau}.
\]

If \(T\), \(h\), \(v_0\), and \(\theta\) are known, the projectile motion equation gives us

\[
g = \frac{2h}{T^2} + \frac{2v_0 \sin \theta}{T} = \frac{2h}{T^2} + \frac{2D \sin \theta}{T\tau}.
\]

The quantities \(T\), \(h\), \(\tau\), \(D\), and \(\theta\) are measured directly.

**How It Works**

The following discussion is not material that you need to learn for 8.01, which is a course in physics, not a course in data analysis. The discussion is only for those who are curious about how the *ProjectileMotion* program estimates the error in the values it calculated for \(g\) and \(r\).
In this experiment, we do not fit a function to our data. We calculate the range \( r \) and the gravitational acceleration \( g \) in terms of five quantities that we measured. All of the measured quantities \( h, \theta, D, T \) and \( \tau \) have some error associated with them, which can be expressed by deviations \( \delta h, \delta \theta, \delta D, \delta T \) and \( \delta \tau \). The deviations are with respect to the average values that are measured and are (reasonably) assumed to be equally likely positive or negative. The question to answer here is: how do these errors produce corresponding errors in the quantities \( r \) and \( g \) that are calculated from them? The answer comes from calculus. If the changes are small, the corresponding change \( \delta r \) in the range \( r \) will be

\[
\delta r = \frac{\partial r}{\partial h} \delta h + \frac{\partial r}{\partial \theta} \delta \theta + \frac{\partial r}{\partial D} \delta D + \frac{\partial r}{\partial T} \delta T + \frac{\partial r}{\partial \tau} \delta \tau .
\]

The quantity \( \frac{\partial r}{\partial h} \) is called a partial derivative. It means that you calculate the derivative with respect to \( h \) assuming that all the other quantities are constants. You will not cover this in a math course until 18.02, but I hope that it makes sense. What we want to calculate is the variance of the range, represented by the symbol \( \sigma^2_r \), and its square root the standard deviation \( \sigma_r \). The variance is defined to be the average of \( (\delta r)^2 \), which is represented as \( \langle (\delta r)^2 \rangle \). (Angular brackets \( \langle \cdot \cdot \cdot \rangle \) are frequently used to represent an average.) Here, you may imagine that the measurements are repeated many times and the results are averaged. If positive and negative deviations are equally likely, then the averages \( \langle \delta h \rangle, \langle \delta \theta \rangle, \langle \delta D \rangle, \langle \delta T \rangle \) and \( \langle \delta \tau \rangle \) are all zero.

The expression to evaluate is therefore

\[
\sigma^2_r = \langle (\delta r)^2 \rangle = \left\langle \left( \frac{\partial r}{\partial h} \delta h + \frac{\partial r}{\partial \theta} \delta \theta + \frac{\partial r}{\partial D} \delta D + \frac{\partial r}{\partial T} \delta T + \frac{\partial r}{\partial \tau} \delta \tau \right)^2 \right\rangle .
\]

The square has to be multiplied out (use the multinomial theorem) and the individual terms averaged. If the errors in the measured quantities are uncorrelated (which seems reasonable here) then the average of cross terms, such as \( \langle \delta h \delta \theta \rangle = \langle \delta h \rangle \langle \delta \theta \rangle \), will be zero. That means that only the squares of the individual terms will survive when the average is carried out, and gives the result

\[
\sigma^2_r = \left( \frac{\partial r}{\partial h} \right)^2 \sigma^2_h + \left( \frac{\partial r}{\partial \theta} \right)^2 \sigma^2_\theta + \left( \frac{\partial r}{\partial D} \right)^2 \sigma^2_D + \left( \frac{\partial r}{\partial T} \right)^2 \sigma^2_T + \left( \frac{\partial r}{\partial \tau} \right)^2 \sigma^2_\tau .
\]

There is still some grubby work to do (calculating the partial derivatives and simplifying the resulting algebra) but this is a recipe to relate the variance in the range \( r \) to the variances in the measured quantities. As \( r \) does not depend explicitly on \( h, \partial r/\partial h = 0 \). After calculation of the other derivatives from

\[
r = v_0 T \cos \theta = \frac{DT \cos \theta}{\tau},
\]

the result is

\[
\sigma^2_r = \left( \frac{DT \sin \theta}{\tau} \right)^2 \sigma^2_\theta + \left( \frac{T \cos \theta}{\tau} \right)^2 \sigma^2_D + \left( \frac{D \cos \theta}{\tau} \right)^2 \sigma^2_T + \left( \frac{DT \cos \theta}{\tau^2} \right)^2 \sigma^2_\tau .
\]
With a little algebra, and using the expression for $r$, this can be written as:

$$
\sigma_r^2 = r^2 \left( \tan^2 \theta \sigma_\theta^2 + \frac{\sigma_D^2}{D^2} + \frac{\sigma_T^2}{T^2} + \frac{\sigma_\tau^2}{\tau^2} \right),
$$

or

$$
\sigma_r = r \sqrt{\tan^2 \theta \sigma_\theta^2 + \frac{\sigma_D^2}{D^2} + \frac{\sigma_T^2}{T^2} + \frac{\sigma_\tau^2}{\tau^2}}.
$$

If we know the variances or standard deviations of the measured parameters, this formula can be used to find the standard deviation of the range, and it is the one used in the computer program.

The program lets you enter values for $\sigma_D$, $\sigma_\theta$ and $\sigma_h$. The times are measured every 200 µs, and the program assumes $\sigma_\tau = 0.0004$ s, $\sigma_T = 0.001$ s. Time is usually the quantity easiest to measure accurately in an experiment. Ball bearing dimensions are very precise and $\sigma_D$ is probably much less than the default 0.01 mm used in the program. The error in $T$ is typically 0.3%, but the error in $\tau$ is more significant, around 3%, and $\sigma_\theta = 2^\circ$ gives about a 2% error. The major sources of error in the determination of $r$ are $\sigma_\tau$ and $\sigma_\theta$.

A similar calculation can be carried out to get $\sigma_g^2$; it is a bit more complicated.

$$
\sigma_g^2 = \langle (\delta g)^2 \rangle = \left\langle \left( \frac{\partial g}{\partial h} \delta h + \frac{\partial g}{\partial \theta} \delta \theta + \frac{\partial g}{\partial D} \delta D + \frac{\partial g}{\partial T} \delta T + \frac{\partial g}{\partial \tau} \delta \tau \right)^2 \right\rangle.
$$

Again, assuming uncorrelated errors in the measured quantities,

$$
\sigma_g^2 = \left( \frac{\partial g}{\partial h} \right)^2 \sigma_h^2 + \left( \frac{\partial g}{\partial \theta} \right)^2 \sigma_\theta^2 + \left( \frac{\partial g}{\partial D} \right)^2 \sigma_D^2 + \left( \frac{\partial g}{\partial T} \right)^2 \sigma_T^2 + \left( \frac{\partial g}{\partial \tau} \right)^2 \sigma_\tau^2.
$$

where

$$
\frac{\partial g}{\partial h} = \frac{2}{T^2}, \quad \frac{\partial g}{\partial \theta} = -\frac{2D \cos \theta}{T \tau}, \quad \frac{\partial g}{\partial D} = \frac{2 \sin \theta}{T \tau}, \quad \frac{\partial g}{\partial T} = -\frac{2D \sin \theta}{T^2 \tau}, \quad \frac{\partial g}{\partial \tau} = \frac{4h}{T^3} - \frac{2D \sin \theta}{T^2 \tau}.
$$

Doing the algebra gives

$$
\frac{\sigma_g^2}{4} = \left( \frac{h}{T^2} \right)^2 \left( \frac{\sigma_h^2}{h^2} + 4 \frac{\sigma_D^2}{T^2} \right) + \left( \frac{D \cos \theta}{T \tau} \right)^2 \sigma_\theta^2 + \left( \frac{D \sin \theta}{T \tau} \right)^2 \sigma_T^2 + \left( \frac{\sigma_D^2}{D^2} + \frac{\sigma_T^2}{T^2} + \frac{\sigma_\tau^2}{\tau^2} \right).
$$

In this case, $\sigma_h$ is also explicitly a significant source of error. This is the formula the program uses to find the standard deviation $\sigma_g$.

Experiment 03 8 September 26, 2005