Purpose of the Experiment:
In this experiment you will study the motion of a cart on a track in response to an applied force.

- You should be able to test Newton’s law $F = ma$ under real-world conditions. You will see how friction and the inability to measure with arbitrary accuracy affect your ability to test this law.

Setting Up the Apparatus:
You will use a PASCO cart, with a force sensor fastened to it, rolling on a track. The force sensor will let you measure the force applied to the cart and a motion sensor will let you measure its position as a function of time.

First attach an adjustable end stop and a pulley to the end of the track as shown in the photo below. The adjustable stop is to keep the cart from crashing into the pulley.

Adjust the leveling screw at the other end end of the track to get the track as level as you can; test by checking that the cart does not have a greater tendency to roll in one direction than another. A 50 gm brass weight and its 5 gm plastic holder should be fastened to one end of a string that passes over the pulley and attaches to the hook on the force sensor. Place two 250 gm weights in the cart. Set the pulley height so the string from the hook to the pulley is horizontal, and center the pulley so the string is parallel to the track. After you have finished the calibration (page 3), you should adjust the string length between the hook and weight so that the falling weight hits the floor when the end of the cart closest to the motion sensor is between the 28 and 30 cm marks of the scale on the side of the track.
The motion sensor should be placed at the other end of the track, against the stop that is permanently fastened to the track, as in the photo at the left. The apparatus is now ready to use, so you may start the LabVIEW program `ForceMotion.vi`.

It will look very similar to the 8.01 LabVIEW programs you have used before and is controlled by a main pull-down menu above the graph to the left. There is a new item at the bottom of the menu that will be explained at the end of these notes.

For this experiment, make sure the sample rate is set to 100 Hz and the Gain to 100X (that makes the full-scale force sensitivity equal to 0.625 N).

There is also a pull-down menu to control data plotting, to the right above the graph.
The motion sensor connects to digital channels 1 (yellow plug) and 2; the force sensor connects to analog channel A.

Calibration
In this experiment, we are trying to get the last bit of accuracy from the equipment, so the sensors should be calibrated. To do that, click the Calibrate tab. Roll the cart down the track so it is against the adjustable stop. Tare the force sensor and then suspend the 55 grams from the hook and over the pulley. The 55 grams weigh 0.539 N, so adjust the slider so the weight reads 0.539 N. Then measure the distance from the motion sensor element to the nearest end of the cart and adjust the sound speed slider to get the distance you measured. That should complete the calibration, so click one of the other tabs and adjust the string length as as described at the bottom of page 1.

Measuring
The zero of the force sensor tends to drift, so it is important to tare it before each measurement; after taring the force sensor pull the cart down the track, but no closer than 15 cm to the motion sensor. Choose Measure from the main menu and the RUN button will glow bright green. One member of the team should guide the cord to the force sensor to minimize the force it exerts on the moving cart. As soon as you click the button (or type the Esc key) the program will start to measure the cart position and the force on the cart, and will continue to do so until the cart has moved to be about 80 cm from the motion sensor. (If it does not stop automatically at this point, click the red STOP button or type the Esc key to stop it.)

The best way to make a measurement is to click START, hold the cart still for about a second (so as to measure the force on the cart when it is at rest), and then release it. You should see a graph of the raw data like the one on the previous page. On this graph, the upper curve is the echo delay (in ms) of the ultrasonic pulse returned to the motion sensor and the lower curve is 100 times the output voltage of the force sensor. These are the quantities the program measures, but they are not the ones you want to know.

Here is where the plot control menu comes into action. The options are shown to the right. You can plot the cart position, velocity, or acceleration as a function of time, and the last option is to plot the force on the cart as a function of time. You can also return to the raw data plot if you wish. After you have chosen what you want to plot from the menu, you must still ask the program to make the plot.

That can be done either with the Plot Data option from the main menu or by clicking the Replot button just below the Plot Control menu. When you do that, the plot of your choice will appear on the graph and the table in the Results tab will contain the numbers that are plotted on the graph.

To continue the data analysis, the program can carry out a least squares fit by three possible functions: \( y = A \) (Constant), \( y = A + Bx \) (Linear), or \( y = A + Bx + Cx^2 \) (Quadratic), where the \( x \) variable is always the time. You choose the fit from the Fit Function? pull-down menu on the Results tab. When you do the fit (Fit Plotted Data from the main menu) the data that are plotted on the graph will be fit, but you must also choose the range of data that will be included in the fit. Only the data points between the red and blue cursors on the graph will be fit, giving you a lot of flexibility.
To use the cursors, the left (cursor) button on the graph control palette must be selected. You can read the numerical cursor positions at the top of the graph. Position the cursors by dragging them.

As an example, if you want to know the force on the cart before the cart starts to move, on the Force plot you can set the cursors to include only these data and choose a Constant fit. (Fitting a constant averages the data points.) My result is shown below.

![Graph showing force over time with fitted constant line]

The numerical results of the fit are shown on the Results tab. My fit results gave the force as $0.534 \pm 0.004 \text{ N}$, using an input $\sigma_F = 0.03 \text{ N}$ for the force. However, the value of $\chi^2$ was very small (about 0.006) and the Root MSE was also small (0.002).

This illustrates something to watch out for when doing fits. The standard deviation calculated by the fit comes from changing the fit parameter to increase the total squared error in the fit by $\sigma^2$ (i.e., increasing the Root MSE by $\sigma/\sqrt{N_{dp}}$). This means that the standard deviations calculated for the fit coefficients will only be meaningful if the standard deviation for the measured data (here, $\sigma_F$) is reasonably close to correct.

Since I used the manufacturer’s $\sigma_F = 0.03 \text{ N}$, the result of fitting by a constant (essentially averaging the $N_{dp} = 70$ points I fit) to get a force $0.534 \pm 0.004 \text{ N}$ seems reasonable. Why then was $\chi^2$ so small? The Root MSE and $\chi^2$, when fit to the constant term $A$ here, are sensitive only to the fluctuations in the input data. If there is a systematic error in which all of the $N_{dp}$ data points are in error by the same amount, Root MSE and $\chi^2$ will be smaller than they should be. To get $\chi^2 \approx 1$ would require $\sigma_F \approx \text{Root MSE}$ and that is much too small according to what we know about the PASCO force sensor.

Thus it is important to use sensible values for the standard deviation of your measured data in order to get sensible values for the standard deviations of the fit parameters. If the resulting $\chi^2$ seems too small, that suggests systematic errors in the measured data.
As you can see from your graph, the force measurements get noisier as the cart starts to move; that comes from vibration as the cart rolls along the track. If you look at the full graph of the force, you may also see it drop to zero when the weight reaches the floor as well as the collision with the adjustable stop. When I fit the force data for the period when the cart was rolling, it seemed to be constant, although noisy, with a value $0.485 \pm 0.003 \text{ N}$. The force decreased from the “at rest” value because some of the force of gravity was being used to accelerate the falling weight and overcome friction as the string passed over the pulley.

To continue analysis of the measurements, you must find the acceleration. I found the easiest way to do that was to fit a plot of velocity as a function of time to the linear function $A + Bt$.

\[
\begin{align*}
\text{From the slope of the my graph, I found an acceleration } & 0.413 \pm 0.003 \text{ m/s}^2 \text{ (with } \sigma_x = 0.002 \text{ m).} \\
\text{You should do a similar fit to your data. Then you can fill in your results in the table below (this table is repeated in the report form for this experiment).}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Force (at rest, N)</th>
<th>Force (rolling, N)</th>
<th>Acceleration (rolling, m/s$^2$)</th>
<th>Mass (total, kg)</th>
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where “Force” is the force as measured by the force sensor and averaged by fitting to a constant, “Acceleration” is obtained by fitting the velocity plot to $A + Bt$, and “Mass” is the total mass: the cart ($0.250 \text{ kg}$), force sensor ($0.339 \text{ kg}$), the weights in the cart, and the falling brass weight and its holder ($0.055 \text{ kg}$). With the Force and Acceleration include the standard deviations obtained from the fits.
With the numbers in this table, you can use Newton’s second law to estimate the friction when the cart is rolling. For example, because of friction the total mass times the acceleration will be less than the force on the cart when it was at rest. The mass of the cart times its acceleration will be less than the force applied to it when it is rolling because of the need to overcome friction. The report for this experiment asks you to estimate some of the friction forces.

**Data Massaging**

This is an optional discussion for those who may be curious about the pull-down menus at the top left of the Results tab.

The program measures only the distance from the cart to the motion sensor, yet we also want graphs of the velocity and acceleration of the cart. A simple point by point numerical differentiation of the distance data introduces a lot of uncertainty and noise. Doing it twice to get the acceleration would give a plot that looked like a random distribution of points. One could fit the position vs. time curve to an expression like $x_0 + v_0t + \frac{1}{2}at^2$, but this would give only the average velocity and acceleration.

A method to obtain derivatives of data that have some noise, and to reduce some of the noise itself, was introduced by two scientists at the Perkin Elmer Company in the 1960s [A. Savitzky and M. J. E. Golay, Analytical Chemistry 36, 1627-1639 (1964)]. The method is to carry out a local least squares fit of a polynomial to just a few data points near the time you are interested in, and then find the local derivatives from the fit coefficients. This LabVIEW program does that in order to make the velocity and acceleration plots. Even with this approach, the acceleration graph is rather noisy.

Using the Savitzky-Golay analysis, one can make two choices: (i) how many data points to include in the local fit, and (ii) what degree of polynomial to use in the fit. The defaults used in the program are 13 points (i.e., the point where you want the derivative and 6 points before and after it) and a quadratic polynomial—which can give first and second derivatives.

The pull-down menus allow other choices for anyone who is inclined to experiment. The coefficient of the constant term in the local fit represents a local average and provides smoothing of the original data. The force data are rather noisy, and you may apply this smoothing to them. Normally the “raw” force data are plotted on the graph, but a pull-down menu allows plotting of the smoothed data instead.

You may want to experiment with this data manipulation; if you make any changes in the Savitzky-Golay pull-down menus, or even just the standard deviation for the position ($X$) measurements, choose the bottom item on the main menu (Recalculate $V$, $A$) in order to re-do the Savitzky-Golay calculation. Incidentally, whether you fit the raw or smoothed force data, you should get the same results.