Purpose of the Experiment:

In this experiment you allow two carts to collide on a level track and run into a spring that is attached to a force sensor. You will measure the position and velocity of the first cart and the force exerted by the spring while it is compressed. You can analyze your data to determine the following things from this experiment:

- An experimental test of the conservation of momentum in elastic and inelastic collisions.
- Determination of the maximum kinetic energy that is lost to non-conservative work in a completely inelastic collision.
- Because you can only measure the position of one cart and the force exerted by the spring, you will analyze the one dimensional collisions in order to determine values that you cannot measure directly.

As the track is level, we do not need to consider changes in the gravitational PE of the carts and may take it to be zero. If we consider the two carts to be an isolated system (this is a good approximation so long as the friction forces are small), the only mechanical energy is kinetic, $K$. A collision is inelastic if some of the initial $K$ is lost to non-conservative work.

A collision is elastic if no $K$ is “lost.” A collision is completely inelastic if the maximum amount of $K$ (consistent with conservation of momentum) is converted to non-conservative work; this happens when the two colliding objects stick together. Sometimes that is what people mean when they refer simply to inelastic collisions.

Setting Up the Experiment:

- At the end of the track with the level adjustment screw attach a force sensor with the hook replaced by the lighter of the two springs.
- Clip the motion sensor to the other end of the track.
• Level your track as well as you can using the level adjustment screw. Test by making sure an empty cart does not have a tendency to roll in either direction; the test is more sensitive if you put two 250 gm weights in the cart.

The LabVIEW Program

The LabVIEW program for this experiment is Momentum.exe; it is very similar to the program you used last week for work/energy. The main pull-down menu is slightly different, with “Analyze Data” being the command to fit data or calculate the integral under a curve. Besides calculating the integral, the program can fit the functions $A$, $A + Bx$ or $A + Bx + Cx^2$ to the data. The position of cart A and the force on the spring are measured by the program.

Connect the motion sensor (yellow plug into jack 1) and the force sensor to the SW750 interface. The force sensor should be plugged into channel A of the SW750. Be sure to tare the force sensor before each measurement.

For all of the measurements set the Run Time to 10 s, the SampleRate to 100 Hz and the Gain to 10X. When you choose Measure from the pull-down menu the RUN button changes to bright green. Clicking the button (or typing the Esc key) starts the measurement; the button will change to red and say STOP. Measurement will stop when you click the button or the Run Time has passed. While measuring the cart position and the force, the program will not actually record the data until the distance to the cart crosses a trigger value of 30 cm while increasing, and recording will stop at the same trigger value with x decreasing. The normal way to make a measurement is to begin with the cart from 16 to 20 cm in front of the motion sensor, click the RUN button, and push the cart. The cart will roll at least 10 cm before data are recorded; that gives the cart time to stabilize and roll smoothly after you push it. The program will record measurements of $x$ at 100/sec and force at 1000/sec until the cart returns to the 30 cm point or the Run Time elapses.

Completely Inelastic Collisions:

You should study completely inelastic collisions first in this experiment. Place two empty carts on the track with the Velcro pads facing each other. One cart (which I call the target cart, with mass $m_g$) should be placed on the track with the end with the Velcro pads about 70 cm from the motion sensor; then the other end of the cart will be about 10 cm
from the spring on the force sensor. The second cart (which I will call the incident cart, with mass $m_A$) should be placed on the track between 16 cm and 20 cm from the motion sensor. You should push the incident cart just hard enough that it comes back to the starting point after colliding with the other cart and bouncing off the spring. If you push too hard a cart may jump during the collision, and if you push too softly cart A will not come back far enough to stop the program from taking data.

You should experiment to get this right before you start to make measurements. How hard you need to push cart A changes with the masses of the two carts and whether or not the collision is inelastic, so you will have to find this out by trial and error for each of the measurements that you make.

Notation: the subscripts A and B refer to the carts, and subscripts 1 and 2 are before and after the carts collide, respectively. The carts have a mass of 0.250 kg. Make measurements with $m_A = m_B = 0.250$ kg (both carts empty), and $m_A = 0.500$ kg, $m_B = 0.250$ kg (add one .250 kg weight to the incident cart A). Measure $v_{A,1}$ before the carts collide and $v_2 = v_{A,2} = v_{B,2}$ after the collision when the two carts are stuck together but have not yet hit the spring.

The raw data when $m_A = m_B = 0.250$ kg are shown in the plot below.

The best way to determine $v_{A,1}$ is to plot Position ($x$ vs. $t$) and do a Linear fit to $x(t)$ before the collision. The data between the cursors will be included in the fit; there is some rounding of the $x$ vs. $t$ during the collision, so don’t go right up to the collision with your fit.
Find $v_2$ by a fit to data after the collision of the carts. Be careful not to include any data from the region where the carts are compressing the spring; a Force plot can show you the time when this begins to happen.

Repeat the inelastic collision measurements for $m_A = 0.500$ kg with $m_B = 0.250$ kg. Record the values of $v_{A,1}$ and $v_2$ for each collision in the table below. Calculate the kinetic energies $K_{A,1}$ and $K_{A,2}$. 
Elastic Collisions:

These measurements will be done exactly like the preceding ones except the carts are placed on the track with the opposite ends (the ones containing magnets) facing each other. Then the collision between the carts will be elastic. You should measure collisions with \( m_A = m_B = 0.250 \text{ kg} \) (both carts empty), and \( m_A = 0.750 \text{ kg} \), \( m_B = 0.500 \text{ kg} \). Measure \( v_{A,1} \) (before) and \( v_{A,2} \) (after the carts collide, but before the target cart bounces back to hit the incident cart).

Note: because of the “softness” of the collision between the carts (see the appendix on the next page) it is especially important to avoid regions where \( x \) vs. \( t \) shows any curvature when you do the fits to determine \( v_{A,1} \) and \( v_{A,2} \). The motion sensor can only measure the velocity of the incident cart. You know the initial velocity of the target cart, \( v_{B,1} \), is zero, and you can find \( v_{B,2} \) after the collision from the impulse given to the target cart by the spring.

If you assume that cart B collides elastically with the force sensor, then during the collision the momentum of cart B changes by \( \Delta \mathbf{p}_B = -2m_B \mathbf{v}_{B,2} \). This change in momentum is the impulse that the force sensor exerts on the target cart B. Cart B therefore exerts an equal and opposite impulse on the force sensor; both have magnitude \( I = 2m_B v_{B,2} \). Measuring this impulse allows you to calculate the velocity of the target cart after the collision. Here is how to measure the impulse.

<table>
<thead>
<tr>
<th>( m_A )</th>
<th>( m_B )</th>
<th>( v_{A,1} ) [m·s(^{-1})]</th>
<th>( v_2 ) [m·s(^{-1})]</th>
<th>( K_1 ) [J]</th>
<th>( K_2 ) [J]</th>
<th>( \Delta K ) [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250 kg</td>
<td>0.250 kg</td>
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<tr>
<td>0.500 kg</td>
<td>0.250 kg</td>
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Make a Force plot. Drag the cursors on to the force peak (where cart B is compressing the spring) and expand the \( x \) scale so the peak fills the graph. Set the cursors to the start and end of the peak. Choose “Integral” from the Fit Function? pull-down menu on the Table tab and type 0.1N into the F StdDev on the tab. Then choose Analyze Data from the main pull-down menu. That will calculate the force-time integral between the cursors and fill in the area with gray on the plot. (The numerical value of the integral and an estimated standard deviation are given on the Table tab.) This integral is the impulse \( I \) given to cart B during its collision with the spring; from \( I \) you can calculate \( v_{B,2} \).
Carry out these measurements and analysis for the two elastic collisions and enter your results in the table below. (This table and the table on page 4 are reproduced in the report for this experiment.)

<table>
<thead>
<tr>
<th>( m_A )</th>
<th>( m_B )</th>
<th>( v_{A,1} ) [m·s(^{-1})]</th>
<th>( K_1 ) [J]</th>
<th>( v_{A,2} ) [m·s(^{-1})]</th>
<th>( I ) [N·m]</th>
<th>( v_{B,2} ) [m·s(^{-1})]</th>
<th>( K_2 ) [J]</th>
<th>( \Delta K ) [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250 kg</td>
<td>0.250 kg</td>
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<tr>
<td>0.750 kg</td>
<td>0.500 kg</td>
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</table>
Appendix: Magnetic Force

This is not part of the experiment you are asked to do, but the result has been used on problem sets from previous years. When the carts collide elastically, they exert forces on each other by the repulsion between two pairs of magnets. I was able to measure the force law between the two carts by replacing the spring on the force sensor with a pair of magnets and rolling one cart to collide with the force sensor. (I used a different LabVIEW program to fit an exponential function to the result.) Here is my result.

The solid line is a fit by the function $F = Ae^{-x/L}$. There is some systematic deviation visible, but it is a pretty good approximation to the force law between the carts as they collide, and is the basis of the homework problem on the next page.