Experiment 3: Momentum and Collisions

Purpose of the Experiment:
In this experiment you collide a cart with a spring that is attached to a force sensor and also collide one moving cart with a second initially stationary cart on a level track. You will measure the velocity of one cart and the force exerted by the spring while it is compressed. You will analyze your data to investigate the following things from this experiment:

- The correspondence between impulse and change in momentum.
- The predictions derived using conservation of momentum in inelastic collisions as well as momentum and energy for elastic collisions.

When a net external force acts on a moving object, it will change the momentum of the object. The basic equation can be written in two equivalent ways: \( \frac{d\vec{p}}{dt} = \vec{F} \) or \( \Delta \vec{p} = \int \vec{F} dt \), where \( \Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} \). You will test the integral formula by measuring both the velocity before and after a spring acts on a cart and the integral of the force of the spring as a function of time. The computer program will integrate the force data numerically.

The force sensor with a spring is mounted at one end of the track and the motion sensor is clipped to the other end. To balance all external forces along the track, it should be level, but since you will measure speed just before and just after collisions, it is not critical that the track be perfectly level.

When no net external force acts on an object or group of objects, the total momentum is a constant. So-called “internal” forces, such as those between objects in a collisions, do not change the total momentum. The collisions on the track are one dimensional so we only need to worry about one component of the momentum vector. If the track is level and friction can be neglected, there are no net external forces. If a moving cart hits a stationary cart and the two stick together after the collision, the relevant momentum equation is \( m_1 v_0 = (m_1 + m_2) v_1 \). If a moving cart hits a stationary cart and the two do not stick together after the collision, the relevant momentum equation is \( m_1 v_0 = m_1 v_1 + m_2 v_2 \) which has two unknowns, so you need more information to solve the problem. If the collision is also known to be elastic (i.e. KE is conserved), the additional equation is \( \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \). These two equations can be combined and solved to find \( v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_0 \) and \( v_2 = \frac{2 m_1}{m_1 + m_2} v_0 \). In this experiment, you will measure the velocity of the incoming cart before \( (v_0) \) and after \( (v_1) \) inelastic and elastic collisions and attempt to verify two of these three equations for final velocities. If you have time left, the optional part also checks the equation for the speed of the struck cart \( (v_2) \) in an elastic collision.
Data is taken with the Momentum program. Like our other LabVIEW programs, it is controlled by a pull-down menu above the left side of the graph. You will use the “Measure” and “Integrate Data” functions. There is also a pull-down menu to control plotting of data.

**Momentum and Impulse:** In this part, you push one cart so it bounces off of the spring.

**Be sure to tare the force sensor before each measurement.** You can find the speed of the cart before and after it hits the spring by selecting Velocity from the plotting pull-down menu and hitting “Replot Data”. You should see something like the figure at the right. You can move the cursor to find the speed just before (see blue cursor) and just after (see red cursor) the cart hits the spring as shown. It might help to expand the scale.

To measure the integral of Force versus time (i.e. the impulse), use the plotting controls to look at the force plot. Move the cursors to either side of the peak, go to the “Table” tab and make sure the fit type is Integral (see upper left oval on the right figure below). Under the Action Menu, select “Integrate Data” and read off the result from the integral box (see lower right oval on the right figure below). An expanded version of the force plot after the fit is shown in the left figure below with the area integrated shaded in grey.

**Collisions**

**Don’t let the rebounding carts hit the motion sensor! HANDS OFF!!**

For inelastic collisions, arrange the carts with the Velcro pads facing each other. You may decide to put weights on one or both of the carts. If you push too hard a cart may jump off the track during the collision. You should experiment to get this right before you start to make measurements. For inelastic collisions, your velocity plot should look something like the plot at the right. Use the cursor to read off the velocities just before (see blue cursor near time=1.35 seconds on the figure) and just after (see red cursor) the collision.
For elastic collisions, arrange the carts with the blank sides facing each other. The carts have opposing magnets so they repel each other. You need to decide what weights to put on one or both of the carts. In the first part, the incoming cart should be lighter and in the second part, it should be heavier. As before, the target cart should be placed on the track with the end facing the incoming cart about 50-60 cm from the motion sensor. For the lighter incoming cart, your velocity plot will look similar to the bouncing off the spring experiment (see figure at the top of page 2), while for the heavier incoming cart it will resemble the inelastic velocity plot (see figure at the bottom of page 2). Use the cursor to read off the velocities of the incoming cart just before and after the collisions.

Elastic Collision: Additional Analysis
Under the assumption that the cart hits the spring and rebounds with the same speed (which you can evaluate and perhaps correct for using your data from the first part of this experiment), the change in momentum of the struck cart can be used to calculate the speed from: \[ \int F \, dt = 2m_2v_2, \] where \( \int F \, dt \) is the magnitude of the impulse (i.e. the integral of Force versus time). Measuring this impulse allows you to calculate the velocity of the struck cart after the collision. The speed of \( m_2 \) after the collision can be used to study these questions:

While the carts were colliding, did the momentum change?

While the carts were colliding, did the total kinetic energy change?

Appendix: Magnetic Force
This is not part of the experiment you are asked to do, but the result is interesting. When the carts collide elastically, they exert forces on each other by the repulsion between two pairs of magnets. The force function between the two carts was found to look like the plot at the right. The solid line is a fit by the function \( |F| = Ae^{-x/\ell} \), where \( x \) is the separation between the carts and \( \ell \) is a characteristic length associated with the magnets. Note that this force is very different from an ideal compressed spring, \( |F_{spring}| = k|\ell - \ell_0| \).

There is some systematic deviation visible, but it is a pretty good approximation to the force law between the carts as they collide.

If we define the positive \( \hat{x} \) direction to point in the direction of the magnet force, then it is given by \( \vec{F}_{mag} = Ae^{-x/\ell} \hat{x} \), and an external force of \( \vec{F}_{Ext}(x) = -Ae^{-x/\ell} \hat{x} \) is required to push the carts together. If the carts start very far apart (an infinite distance), then the work done by that external force to bring them to a separation distance \( d \) is:

\[ W = \int_{\infty}^{d} F_{Ext}(x) \cdot dx = -A \int_{\infty}^{d} e^{-x/\ell} \, dx = A\ell e^{-x/\ell} \bigg|_{\infty}^{d} = A\ell e^{-d/\ell} \]

You can check this result by taking the derivative of the next to last term. Note that since the carts repel each other, you need to do positive work to push them together. This might make a good exam question ☺.
Experiment 3: Momentum and Collisions

Electronic setup:

1. The 750 interface should be powered on (green light comes on).
2. Connect the motion sensor (yellow plug into jack 1, black into 2) and the force sensor (channel A) to the SW750 interface.
3. Be sure to tare (zero) the force sensor before each measurement (button on the side).
4. Download and run the Momentum program. Do NOT run it from the web, download it to your desktop first.
5. If you get a warning question about verifying the publisher of the program, click “Run”. If you get a different error, something is turned off or unplugged, so you need to check your setup first. We are L10.

Mechanical setup:

1. The motion sensor should be aimed slightly below the center of the cart rather than pointing directly at it.
2. The slide switch on top of the motion sensor should be set to the narrow beam position.
3. Your track should be level. If necessary, you can improve it using the level adjustment screw. The best way to test this by looking at the velocity data for a cart running from one end of the track to another. Ideally, the velocity will be a constant. Since we will be measuring speed just before and just after collisions, it is not that critical that the track be perfectly level.

To take data:

1. Like our other LabVIEW programs, this one is controlled by a pull-down menu above the left side of the graph. You will use the “Measure” and “Integrate Data” functions. There is also a pull-down menu to control plotting of data.
2. When you are ready to measure, hold the cart or carts in position and choose Measure from the pull-down menu. The RUN button will change to bright green. Click the RUN button and launch the incoming the cart.
3. NOTE: The program starts taking data when the cart is about 30 cm from the motion sensor and moving away from it so you can hit “Run” before releasing the cart.
4. If you wave your hands around the carts as they move, you may confuse the motion sensor. HANDS OFF!!

Momentum and Impulse:

1. Don’t let the rebounding cart hit the motion sensor! HANDS OFF!!
2. In this part, you push one cart so it bounces off of the spring. Do not put weights on the cart.
3. Be sure to tare the force sensor before each measurement.
4. To find the speeds, select Velocity from the plotting pull-down menu and hit “Replot Data”. You should see something like the picture in the writeup (top of page 2).
5 Move the cursor to find the speed just before and just after the cart hits the spring. It might help to expand the scale.

6 If you have forgotten how to expand the scale and move the cursors, see the end of the writeup for previous experiments (linked from the 8.01L Experiment page).

7 Now, you need to measure the integral of the Force versus time plot. First, use the plotting controls to display the force plot.

8 Move the cursors to either side of the peak, go to the “Table” tab and make sure the fit type is Integral. Under the Action Menu, select “Integrate Data” and read off the result from the integral box (see drawing in writeup).

9 An expanded version of the force plot after the fit is shown in the writeup with the integrated area shaded in grey.

Inelastic Collision

1 Don’t let the rebounding carts hit the motion sensor! HANDS OFF!!
2 Arrange the carts with the Velcro pads facing each other. You may decide to put weights on one or both of the carts.
3 The target cart should be placed on the track with the end with the Velcro pads about 50-60 cm from the motion sensor.
4 If you push too hard a cart may jump during the collision. You should experiment to get this right before you start to make measurements.
5 An example of what your velocity plot should look like is in the writeup (bottom of page 2).
6 Use the cursor to read off the velocities just before (see blue cursor near time=1.35 seconds on the figure) and just after (see red cursor) the collisions.

Elastic Collision

1 Don’t let the rebounding carts hit the motion sensor! HANDS OFF!!
2 Arrange the carts with the blank sides facing each other. The carts have opposing magnets so they repel each other.
3 You need to decide what weights to put on one or both of the carts. In the first part, the incoming cart should be lighter and in the second part, it should be heavier.
4 The target cart should be placed on the track with the end facing the incoming cart about 50-60 cm from the motion sensor.
5 If you push too hard a cart may jump during the collision. You should experiment to get this right before you start to make measurements.
6 For the lighter incoming cart, your velocity plot will look similar to the bouncing off the spring experiment, while for the the heavier incoming cart it will resemble the inelastic collision velocity plot.
7 Use the cursor to read off the velocities just before and after the collisions.

Optional: Additional Elastic Collision Analysis:
If you finish early, you can do additional analysis of an elastic collision by using the spring impulse to find the speed of the second cart after the collision. See writeup for details.