Experiments 05 and 06: Angular Momentum

Purpose of the Experiments:
In these experiments you investigate rotational collisions and the conservation of angular momentum in rigid body rotational dynamics. It is the rotary counterpart of linear collisions.

The heart of the experiments is a high quality DC motor to spin a rotor up to several hundred radians per second. When power to the motor is shut off, it serves as a tachometer-generator whose output voltage is proportional to the angular velocity of the rotor; thus the angular velocity of the rotor can be determined by measuring the output voltage. When you hold down the red pushbutton switch on the apparatus, power is applied to the motor; when you release it, the rotor coasts and the output voltage can be read by the computer. These experiments will give you experience in

- measuring and calculating moments of inertia,
- calculating rotational kinetic energy and non-conservative rotational work, and
- using several other concepts from our study of rotational dynamics, including the conservation of angular momentum, conservation of rotational kinetic energy and the kinetics of rotational motion when the angular acceleration is constant.

Setting Up the Experiments:
Plug the rotary motion apparatus into its power supply; you should see the LED in the plastic pipe elbow come on.
In Experiment 05 you will calibrate the equipment and measure the moment of inertia of the rotor. Next week in Experiment 06 you will use the calibrated apparatus to study rotational collisions.

At the start of Experiment 05, use two voltage sensor plugs to connect the output from the apparatus to inputs A and B on the SW750 interface box. Connect the generator output voltage (the jacks farthest from the power input connector) to input A and the phototransistor output to input B.

The data acquisition and analysis for both experiments will be done with the LabVIEW program called AngularMomentum.exe.

The program operation is controlled by the main pull-down menu at the upper left of the graph and the three tabs labeled “Graph”, “Run and Fit Parameters” and “Calibrate” on the front panel. The Calibrate tab is an addition to what you have become used to our previous LabVIEW programs. It is there because two calibrations must be done in this experiment. Before you can carry out any rotational collision measurements, you must first calibrate the tachometer-generator and then measure the moment of inertia of the rotor. These are the tasks of Experiment 05; in Experiment 06 you will study rotational collisions.

The first calibration is to calibrate the tachometer generator output, i.e., what is $\omega$ for 1 V output? When the Calibrate tab is active (that means it’s the one whose contents you can see) and the “Which Calibration?” pull-down menu on the tab is set to Tachometer, the program will measure the voltages on channels A and B and assume that data files saved by or read into the program contain two voltages as a function of time. Also, the Fit Data option of the main pull-down menu will use these two voltages to calibrate the generator, in a way that you will see presently.

Otherwise—when the Calibrate tab is not active (visible) or it is active with the pull-down menu on the tab set to Moment of Inertia—the program measures only the voltage on input channel A and uses the generator calibration result to convert the voltage to angular velocity; then the Fit Data option will fit $\omega$ to $A + Bt$; and data files saved by or read into the program will contain only angular velocity as a function of time.

**Cursor Control:** This gadget above the graph helps you position the cursor more accurately. The button in the middle lets you select which cursor moves. The arrows move the cursor one data point each time you click them. (On Windows machines the keyboard left and right arrow keys do the same thing.)
Part I: Experiment 05

Calibrating the Generator:

Put a black sticker on the white plastic centerpiece of the rotor so that it will be illuminated by the LED; the reflected light will be detected by the phototransistor. (The voltage output will be higher when the light is reflected from the black tape and lower when more light is reflected from the white plastic.)

Set the voltage sample rate to 5000 Hz from the pull-down menu and type a run time of 0.25 (s) into the window. For this calibration you want to measure both the phototransistor output voltage and the voltage from the generator. Then perform the following steps.

1. Click the Calibrate tab and make sure the “Which Calibration?” pull-down menu is set to Tachometer.

2. While the Calibrate tab is still active (visible) choose Measure from the main pull-down menu; the START button will become brighter green, indicating the program is ready to make the measurement.

3. Spin the motor up for several seconds, release the button and allow the motor to coast for about a second, then click the START button (or type the Esc key on the keyboard). The START button will briefly turn red and say STOP while the computer makes the measurement.

After the measurement is over, click the Graph tab and you will have a plot of the two voltages, as in the figure below. The purple curve (Plot 0) is the generator output voltage and the green one (Plot 1) is the phototransistor voltage.
To calibrate the generator, you need to find the time between the first and last fully visible phototransistor peaks on the graph. Here is how to do that.

1. Place one cursor at the center of the first complete voltage peak and the other at the center of the last complete voltage peak of the phototransistor output. You can do this roughly by dragging the cursor onto the peak. Then use the zoom control (center button on the graph palette) to expand each peak in turn and position each cursor more accurately in the center of the peak.

2. Next, count the number of rotation periods between the two peaks; then click the Calibrate tab. You can type in the likely errors in the voltage (Sigma V) and time (Sigma T) measurements, but the default $\sigma_V = 0.01 \text{ V}$ and $\sigma_T = 0.0005 \text{ s}$ are OK.

3. Make sure the pull-down menu on the tab says Tachometer, and choose Fit Data from the main pull-down menu. A dialog window will open so you can type in the number of rotation periods.

As soon as you enter the number of periods, the program will calculate the average generator voltage and the average angular velocity for the time between the cursors and fill in the remaining fields in the box. The results will be saved in the computer’s memory.

All of the fields in the Tachometer calibration box should now have numbers in them:

That completes the tachometer calibration. Enter the results for the calibration factor (Rad/Sec per Volt or $s^{-1} \text{ V}^{-1}$) and its standard deviation in your report for Experiment 05. All future measurements will be only of the generator voltage on input A, so you can remove the cable between the apparatus and input B of the SW750.

If you want to save the raw calibration data, fill in the comment fields on the Run and Fit Parameters tab, then go back to the Calibrate tab (with the pull-down menu on the tab set to Tachometer) before choosing Save Data from the main pull-down menu. It would be a good idea to name the file Calibrate01.txt (rather than Run01.txt) to remind you it is a tachometer calibration data file. If you read the file back into the computer memory, you must do so when the Calibrate tab is active (visible) with the pull-down menu on the tab set to Tachometer.

**Note:** You should write down and keep the calibration for your apparatus (Rad/Sec per Volt) and its standard deviation for use in Experiment 06 next week. The instructors will try to make sure you have the same apparatus and you can just type the numbers you saved into the fields on the Calibrate tab and the program will use them.
Moment of Inertia Calibration
The second calibration you must do is to measure the moment of inertia of the rotating parts of the apparatus. To measure the rotor moment of inertia, a known torque is applied by a falling weight and the angular acceleration is measured. Use a 50 gm brass weight with a 5 gm plastic holder. The apparatus should be set up near the edge of the table, as in the photograph below.

Setting up the string and weight: Tie an overhand or figure-of-eight (better) knot in one end of a string and place it in the kerf cut into the brass washer on the rotor. The string should wrap around the rotor several times, pass over the pulley and be fastened to the weight as shown in the photo. Choose a length of string so that the string has completely unwound and will pull out of the kerf just before the weight hits the floor.

Measuring the moment of inertia: Choose “Moment of Inertia” from the “Which Calibration?” pull-down menu on the Calibrate tab. Set the voltage sample rate to 500 Hz and the run time to 4.0 s. Choose Measure from the main pull-down menu and the START button should light up green. Wind up the string around the rotor so the weight is near the pulley and hold the rotor so it cannot unwind. Click the START button (Esc key) and release the rotor a fraction of a second later. After the 4 seconds have elapsed, click the Graph tab and you should see a graph like the one at the top of the next page.
The theory of the analysis: While the weight is falling, the rotor angular velocity $\omega$ increases ($\alpha > 0$) at a constant rate. After the string pulls out, $\omega$ decreases ($\alpha < 0$) at a constant rate until the rotor stops. The decrease is caused by friction. As the accelerations (slopes of $\omega$ vs. $t$) are constant, the torques must be constant.

Let’s call $\alpha_1$ (a positive number) the angular acceleration while the weight is falling and $\alpha_2$ (a negative number) the angular acceleration while the rotor is coasting to a stop. Let $\tau_f$ be the magnitude of the torque due to friction. During the coasting period, we must have

$$-I_R \alpha_2 = \tau_f$$

where $I_R$ is the moment of inertia of the rotor.

The equations of motion while the weight is falling are a bit more complicated. Suppose $T$ is the tension on the string. The white plastic part of the rotor has a diameter of 1.00 in or a radius of $r = 12.7$ mm. The string will apply a torque to the rotor given by

$$\tau_s = rT.$$ 

A free body diagram of the falling weight will give

$$T = mg - ma$$

where $m$ is the mass of the weight and $a$ is the magnitude of the vertical acceleration of the weight.
Kinematics tells us that

\[ a = r\alpha_1. \]

The final equation we need is (the friction opposes \( \tau_s \))

\[ I_R\alpha_1 = \tau_s - \tau_f. \]

After eliminating \( \tau_f, \tau_s, T \) and \( a \) from these equations, the result is (remember, \( \alpha_2 < 0 \))

\[ I_R = \frac{mr(g - r\alpha_1)}{\alpha_1 - \alpha_2}. \]

**Doing the analysis:** You can obtain \( \alpha_1 \) and \( \alpha_2 \) from the slopes of the two parts of the graph. Both \( m \) and \( r \) are known, and the computer uses this formula to calculate \( I_R \). (The value \( r = 12.7 \) mm is compiled into the program, but you can choose \( m \) to be different from the default 0.055 kg on the Calibrate tab page.)

The program will fit the expression \( \omega = A + Bt \) to all of the data between the cursors when you select Fit Data on the main pull-down menu. To find \( \alpha_1 \), position the cursors on the graph to include data from the region where the weight is falling (\( \omega \) increasing) between \( \omega \simeq 10 \) s\(^{-1}\) and \( \omega \simeq 10 \) s\(^{-1}\) below the peak. Then choose Fit Data from the main pull-down menu. The result of the fit will be drawn in purple over the data; expand the graph with the zoom controls if you want to see more clearly (the graph at the below is an example).

To see the numerical results of the fit, click the Run and Fit Parameters tab. Enter the value for \( \alpha_1 \) into the first table in your report.
To begin finding the rotor moment of inertia, click the Calibrate tab, make sure the pull-down menu says Moment of Inertia and go to the second box (labeled Rotor Moment of Inertia). Make sure the correct mass is entered in the first data field, then click the “Get Alpha” button under the Falling Weight label. That will enter $\alpha_1$ from the fit you just did into the formula for $I_R$.

Return to the Graph tab and place the cursors to select data from the part of the graph where the rotor is coasting ($\omega$ decreasing). Select the data between $\omega \approx 10 \text{ s}^{-1}$ below the peak and $\omega \approx 10 \text{ s}^{-1}$ and do the $\omega = A + Bt$ fit again. Obtain $\alpha_2$ from the Run and Fit Parameters tab and enter it into the first table in your report.

Then return to the Calibrate tab and click the Get Alpha button under the Coasting Rotor label; that will enter $\alpha_2$ into the formula. Complete the calculation by clicking the Calculate button, and the computer will fill in the Moment and Std Dev fields. (The Std Dev is calculated using the standard deviations of $\alpha_1$ and $\alpha_2$ obtained from the fits.)

That completes measurement of the rotor moment of inertia. Enter $I_R$ and its standard deviation into first the table in your report.

Multiply $|\alpha_2|$ by $I_R$ to calculate the friction torque $\tau_f$, and enter $\tau_f$ into the first table in your report.

You should add any comments you want to the text fields on the Run and Fit Parameters tab and save your data (this time as Runxx).

**Measuring the Variation in Friction Torque:**

The friction torque acting on the rotor varies slightly with $\omega$. The variation is not enough to matter for the calibration you have just completed, but it will be significant when you analyze the rotary collisions between washers, where $\omega$ can be much larger than $100 \text{ s}^{-1}$, so it should be measured. This measurement is straightforward. Choose a sample rate of $200 \text{ Hz}$ and a run time of $10 \text{ s}$. Select Measure from the main pull-down menu, run the motor up close to its maximum speed, then start it coasting and click START (or the Esc key) at about the same time. You should get a graph like the one on the next page.

You can see this plot is not linear, that is, $\alpha = -\tau_f/I_R$ depends slightly on $\omega$. Save these data for later use; you can read them back into the program when you need them. When you analyze the slow collision results, you will want to know the average friction torque over a small range of $\omega$, and you can find it from this graph by using the cursors to select the desired range of $\omega$ and fitting only that portion of the graph to $\omega = A + Bt$.

**Note:** If the string did not pull out cleanly or for any other reason you did not get a nice clean result while the rotor was coasting down during the measurement for the rotor moment of inertia, you can get the value for $\alpha_2$ from the graph above. Just set the cursors to restrict the fit to the range $\omega < 100 \text{ s}^{-1}$ and you can find $\alpha_2$ from fitting $\omega = A + Bt$. 

<table>
<thead>
<tr>
<th>Rotor Moment of Inertia:</th>
<th>Fall. Mass (kg)</th>
<th>Fall. Weight</th>
<th>Coasting Rotor</th>
<th>Moment of Inertia</th>
<th>Moment</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0550</td>
<td>Get Alpha</td>
<td>Get Alpha</td>
<td>Calculate</td>
<td>5.03E-5</td>
<td>9.91E-7</td>
<td></td>
</tr>
</tbody>
</table>
To get an idea how much the friction varies with $\omega$, set the cursors to include the top 50 s$^{-1}$ of your graph (i.e., from $\omega = 475$ s$^{-1}$ to $\omega = 425$ s$^{-1}$ on the graph above) and fit this region of data to $\omega = A + Bt$ in order to find $|\alpha|_{\text{fast}}$. Take $\omega_{\text{fast}}$ to be about midway in the range you fit on the graph. Enter these results into the second table of your report. Calculate $\tau_{f_{\text{fast}}}$ and enter that into the table.

Then use the cursors to select the slowest 50 s$^{-1}$ of your graph (i.e., from $\omega = 125$ s$^{-1}$ to $\omega = 75$ s$^{-1}$ on the above graph) and repeat the fit to find $|\alpha|_{\text{slow}}$. Calculate $\tau_{f_{\text{slow}}}$ and enter the results for $\omega_{\text{slow}}$ or $|\alpha|_{\text{slow}}$ and $\tau_{f_{\text{slow}}}$ into the second table of your report.
Part II: Angular Collisions

Setting Up:
Follow the same procedures as last week (pages 1 and 2), with the following differences.

- Connect only the generator output (jacks farthest from the power input connector) to input A of the SW750.

- When you first start the LabView program, click the Callibrate tab and type your calibration results from last week (Rad/Sec per Volt and its standard deviation) into the appropriate fields in the Tachometer Calibration box.

- You may also type your results for the rotor moment of inertia and its standard deviation into the fields of the Rotor Moment of Inertia box. However, this is not necessary as the program does not use these numbers.

- At the bottom of the Calibrate tab is a box labeled Washer Moment of Inertia. If you type the washer dimensions and mass into the fields the program will calculate the washer moment of inertia when you click the Calculate button. (You may also do this calculation with a calculator.)

- To study the collisions, set the Sample Rate to 2000 Hz and the Run Time to 4.00 seconds.

Hints for Successful Collisions:
You will get better results when you drop the washer onto the spinning rotor in the best way. Here is what I found helps.

- Hold the washer level with the hole in the washer centered above the rotor axis and just above the top of the rotor before you drop it.

- Release the washer so it stays level as it falls. This is easier to do if someone else operates the computer.

Velcro:
If you look carefully at the Velcro, you will see it comes in two kinds. One has little hooks and the other is softer and looks more furry. The two kinds stick together when the hooks latch into the “fur.”

The top of the washer that is a permanent part of the rotor has fur. One of the washers you can drop has hooks on one side and fur on the other. The other washer you can drop has hooks on one side and no Velcro on the other side. This makes three kinds of collisions possible. If you drop hooks onto fur the washers stick amost instantly and you get a fast collision. If you drop brass or fur onto fur, the washers slide a while before reaching the same $\omega$ and a slow collision (lasting a few 100 ms) results.

First you should investigate a slow collision, as it is easier to understand the detailed behavior of than the fast collision.
A Slow Inelastic Collision:

Try dropping a washer to produce both brass-on-fur and a fur-on-fur collisions. For each collision you should get a graph something like this one.

Here are the criteria to use in deciding if you have reasonably good data:

- There is a fairly clear break in slope at the beginning and the end of the collision, making it easier to determine the duration of the collision.

- The slope of \( \omega(t) \) is fairly constant during the collision and there is little sign of wobbling of the washer as it falls onto the rotor.

If a brass-on-fur collision seems reasonable, type some information (such as the type of collision and mass of the washer you dropped) into the comment fields of the Run and Fit Parameters tab and save it.

Repeat for a fur-on-fur collision, and save what seems to be a reasonable collision. Then decide which of the slow collisions you want to analyze and begin the analysis. The graph above is a fur-on-fur collision.

**Analysis of the Slow Collision:**

Write the type of collision, \( I_R \) (from last week) and \( I_W \) (for the washer you dropped) in the spaces provided on your report form. From your slow collision graph, find the values for \( \omega_1 \) and \( \omega_2 \) at the beginning and end of the collision, respectively. Enter them into the first table for the slow collision in your report. Also find the times \( t_1 \) and \( t_2 \) for the beginning and end of the collision; calculate \( \delta t = t_2 - t_1 \) (the duration of the collision) and enter it into the table. Calculate the average angular acceleration \( \alpha_R \) of the rotor during the collision. The washer started from rest, so you can also calculate the average angular acceleration \( \alpha_W \) of the washer during the collision. Enter these two quantities into the table.
The next step is to find the frictional torque on the rotor during the collision. Because of friction, the system of the dropped washer and the rotor is not isolated and angular momentum is not conserved. If you know the friction, you can compute its angular impulse during the collision and correct for it.

You recall from last week that the friction depends slightly on the angular velocity. Reload your long coast measurement from last week (page 9) into the computer and set the cursors to fit the data between $\omega = \omega_1$ and $\omega = \omega_2$ for your slow collision. Do the fit and find the magnitude of the average acceleration $|\alpha|_{f\text{coll}}$ during the collision. Compute the average friction torque $\tau_{f\text{coll}}$. Enter these into the second table for the slow collision in your report.

Calculate the angular momentum just before, $I_R \omega_1$, and just after, $(I_R + I_W) \omega_2$, the collision. Enter these in the table. Next calculate the angular impulse $J_{f\text{coll}}$ provided by the friction torque during the collision and enter it into the table.

To complete your analysis of the slow collision answer the questions at the bottom of the first page of the report.

**A Fast Inelastic Collision:**

Keep the same settings for the *LabView* program and this time drop a washer hook side down. You should see a graph like this one.

![Graph](image)
If you expand the $x$-axis on this graph you can see two interesting things.

- The first is that the collision happens quickly. Whatever is going on, it is all over in 30 ms or less. As you can see from your analysis of the slow collision, the impulse of the friction torque is therefore negligible. Thus we can rely on angular momentum being conserved at all times during the collision. Since we can only measure $\omega_R$, this allows us to deduce $\omega_W$.

- Second, there is an interesting dip, where $\omega$ undershoots and then recovers.

It is not difficult to develop a quantitative model for this collision, but it requires an understanding of the behavior of heavily damped harmonic oscillators—which goes beyond what we cover in 8.01.

However we can develop a pretty good qualitative understanding and make a some quantitative statements.

You can find the first clue if you grab the rotor and the washer you dropped onto it and try to rotate them with respect to each other. They will rotate a few degrees and you can feel a restoring force, as if the Velcro were a spring. You can imagine the washer and the rotor rotating with respect to each other in opposite directions, forming the rotational analog of a harmonic oscillator that you could have if two roughly equal masses were attached by a spring.

Of course this will not happen right away. At first the hooked Velcro on the washer slides over the Velcro fur on the rotor, until their angular velocities have become close enough that they can lock together; at that point the Velcro will act like a spring and the washer and the the rotor will form a rotary harmonic oscillator. A graph of $\omega_R$ vs. $t$ might look something like the figure at the left, next page.
The dashed line shows the region where the two parts of the Velcro are sliding over each other; then they lock together and the oscillation begins. All the while, the “center of inertia” (the rotary analog to the center of mass) rotates at the angular velocity $\omega_2$.

What you see on your graph does not resemble the left figure very closely. That is likely for two reasons. First the oscillation is very strongly damped, as in the right figure. Second, the Velcro “spring” probably does not obey Hooke’s law.

Although this is not a quantitative model for behavior of the washer and the rotor during the collision, you can still make some quantitative estimates based on principles such as the conservation of angular momentum.

First, read $\omega_1$ and $\omega_2$ (the values of $\omega_R$ before and after the collision) from your graph and write them into the first table for the fast collision on page two of your report. (Take “after” to be the time when oscillations are no longer visible.) Calculate the angular momentum before and after the collision and put those values in the table as well.

There are four additional questions on page two of the report which you should give quantitative answers to.