Experiment 4: Electrostatic Force

OBJECTIVE

To measure $\varepsilon_0$, the permittivity of free space.

INTRODUCTION

Electrostatic force plays a variety of roles in Nature. Examples include making currents flow in wires, holding atoms and molecules together, making clothes cling, glue stick, etc. The strength of the electrostatic force is proportional to the product of a charge (measured in coulombs) and the electric field (measured in volts/meter) produced by the other charges.

In this experiment, you’ll set up a horizontal parallel plate capacitor and find the voltage at which a piece of aluminum foil of known weight just lifts off the bottom plate. The electric force then just balances the force of gravity:

$$\vec{F}_e = \vec{F}_g$$

(4.1)

In addition to a single thickness of foil, you will also be using folded foils which increases the thickness by $n$ folds. The electric field between the plates is proportional to the surface charge density on the plates. When the electric field is measured in SI units, the constant of proportionality between the field and the surface charge density is determined by $\varepsilon_0$, the permittivity of free space. From the relations between measured or known electrical quantities, material properties and apparatus dimensions you can determine the constant $\varepsilon_0$.

PROCEDURE

To carry out the experiment, you will need the parallel plates that are made from two large washers (2.5 inch diameter). The washers are separated by 3 pieces of insulating perf-board having a thickness of 1.5 mm, at about $120^\circ$ separations, as illustrated in Figure 4.1:
1. Place a small piece of aluminum foil on the lower washer, and apply a voltage generated by HVPS (high voltage power supply) across the washers.

Note: (1) The HVPS has a very high internal resistance so you cannot get a serious shock from it. The experiment depends on the smoothness and cleanliness of the washers and the cleanliness of the foil. This is because charge from the bottom washer must be distributed to the foil.

(2) The foil should be clean and flat, but it also needs to be given a rough texture to allow air to pass under it when it lifts off. This is done by using a piece of Kleenex to press the small piece of foil against a piece of 80 grit sandpaper included with the experiment. Try to handle the foil with your fingers as little as possible to keep it clean. Tweezers work well. You can see (Figures 4.2 and 4.3) that if the foil isn’t flat, either the size of the gap will be reduced or charge will flow out to the part of the foil nearest the upper washer where larger forces will be exerted than if the foil were flat. You will use 3 foils: 0.0003 in, 0.0005 in, and 0.0007 in.

2. Connect the lower washer to the negative side of the HVPS, and the upper washer to the positive side. One multimeter set on the +DC 1000 V scale is used to read the HVPS output by connecting the meter leads across the output of the HVPS (the right multimeter in figures 4.4a and 4.4b). The second multimeter is connected in series with the gap (the left multimeter in figure 4.4a and 4.4b).
3. When the foil lifts and shorts the gap, the second multimeter will register a voltage and the first multimeter will register a drop in the voltage. You want to determine the voltage across the gap just before the foil moves.

- Watch the multimeter readings as you slowly increase the voltage. When the foil lifts and shorts the gap, the right multimeter reading will drop. Record the voltage just before the drop.
You want to equate the gravitational and electric forces on the foil. The gravitational force is just the weight of the foil. The density of the aluminum foil material is \( \rho = 2.7 \) grams/cc \((2.7 \times 10^3 \text{ kg/m}^3)\). The volume of the foil is then its area \( A \) times its thickness \( t \). Therefore the gravitational force is

\[
F_g = |\mathbf{F}_g| = mg = \rho tAg
\]  

(4.2)

Finding the electrical force is a bit subtler. Treat the closely spaced, metal portions of the washers as two equal and opposite parallel disks. The electric force on the foil will be its charge times the electric field it feels. The subtlety lies in calculating that field. The total field in the capacitor is just \( E = \Delta V/d \) where \( \Delta V \) is the applied voltage and \( d \) is the spacing between the plates. But only half that field is due to charges on the upper plate; the other half is due to charges on the lower plate.

In II. Coulomb’s Law - Worked Examples, we have shown that the electric field between two infinite oppositely charged parallel plates is uniform and given by

\[
E = \frac{\sigma}{\varepsilon_o}
\]  

(4.3)

where \( \sigma \) is the charge density \((\text{charge/area})\) on the upper plate. Since the field between the plates is approximated as uniform, the electric potential difference between the plates \((\text{the voltage difference})\) is

\[
\Delta V = Ed
\]  

(4.4)

The charge density \( \sigma \) is therefore

\[
\sigma = \frac{\varepsilon_o \Delta V}{d}
\]  

(4.5)

To find the electric force on the foil, assume that \( \sigma \), the charge density on the foil, is the same as that of the lower washer. Here comes the subtle point. By thinking about the electric field from the charge sheets on the inner surfaces of the plates (positive on top and negative on the bottom) you come to the conclusion that half the total field comes from the upper plate and half from the lower plate. Charges on the foil feel only horizontal forces from other charges on the bottom plate, so the vertical force on the foil is due to the electric field of just the top charge sheet:

\[
F_e = |\mathbf{F}_e| = |Q_{\text{foil}} \mathbf{E}_{\text{top}}| = \sigma A \frac{\Delta V}{2d} = \frac{\varepsilon_o \Delta V^2 A}{2d^2}
\]  

(4.6)
Equating the electrical and gravitational forces:

\[
\frac{\varepsilon_0 \Delta V^2 A}{2d^2} = \rho t A g
\]  
(4.7)

The area \(A\) cancels and we obtain

\[
\Delta V^2 = \left( \frac{2d^2 \rho g}{\varepsilon_0} \right) t
\]  
(4.8)

If you plot \(\Delta V^2\) vs. \(t\), you should get a straight line whose slope is the coefficient of \(t\).
You can calculate the free permittivity of space \(\varepsilon_0\) from your experimental value for the slope:

\[
\varepsilon_0 = \frac{2d^2 \rho g}{\text{slope}}
\]  
(4.9)