**In Class W07D2_2 Solutions: Current Sheet**

**Problem:** Calculate the magnetic field everywhere due to an infinite sheet of current

**Solution:**

As always, the first step is to think about the problem a little. A current sheet is a symmetric source, and will generate a magnetic field up on the right and down on the left, as pictured at left. By symmetry these two field strengths will have the same magnitude (and the field on the center of the sheet, $x=0$, will be zero). We use this symmetry to choose the Amperian loops, and write down Ampere’s law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

There are two regions we need to think about – outside the sheet and inside the sheet. Let’s start outside – pictured at left. We will walk around the loop in the direction of the magnetic field, so that the integral is positive on the sides, and zero on the top and bottom. Starting at the upper left corner then we have:

$$\oint \mathbf{B} \cdot d\mathbf{s} = Bl + 0 + Bl + 0 = 2Bl$$

How much current is enclosed? In this case we are enclosing the entire thickness of the sheet, $2d$, and a length $l$ of it. Thus $I_{\text{enc}} = JA_{\text{enclosed}} = J2dl$ and we can solve for $B$:

$$\oint \mathbf{B} \cdot d\mathbf{s} = 2Bl = \mu_0 I_{\text{enc}} = \mu_0 J2dl$$

$$B = \mu_0 J \hat{x} \text{ in the direction pictured}$$

Inside we will choose a similar loop except that the sides will be inside the sheet, running from $-x$ to $+x$ (with width $2x$ instead of the $w$ indicated above). The integral will be the same because the width doesn’t appear in it. The current enclosed, however, changes to $I_{\text{enc}} = JA_{\text{enclosed}} = J2xl$. Thus we have:

$$\oint \mathbf{B} \cdot d\mathbf{s} = 2Bl = \mu_0 I_{\text{enc}} = \mu_0 J2xl$$

$$B = \mu_0 J \hat{x} \text{ in the direction pictured}$$

We can summarize (I wouldn’t get so fancy on an exam, but just so you know it can be done):

$$\mathbf{B} = \begin{cases} \mu_0 Jx \left( \frac{x}{|x|} \right) \hat{j} & \text{for } |x| < d \\ \mu_0 Jd \left( \frac{x}{|x|} \right) \hat{j} & \text{for } |x| \geq d \end{cases}$$