In Class W10D1_2 Solutions: LR Circuit

**Problem:** Consider the below LR circuit which has been running for a long time and has a current $I_0$ in it. We suddenly short out (turn off) the battery:

1. What direction does the current flow just after turning off the battery (at $t=0+$)? At $t=\infty$?
2. Write a differential equation for the circuit
3. Solve and plot $I$ vs. $t$ and voltmeters (across $R$ and across $L$) vs. $t$

**Solution:**

Inductors try hard to keep the current through them constant. Since our inductor has a current $I_0$ flowing clockwise through it when the battery is suddenly shut off, that current will continue to flow immediately afterwards: **(1) Still Clockwise.**

After a long time the current will decay away to zero, although it will always be clockwise while it flows. Before doing any differential equations, we should think about what is going on in the problem. We know that in these kinds of circuits that all values either exponentially decay to zero or “exponentially decay upwards” to some constant. In this circuit everything will exponentially decay to zero, as the energy which has been stored up in the inductor decays away and it is no longer able to push a current. The voltage across the resistor is simply proportional to the current ($V=IR$) so it decays, and the voltage measured across the inductor, which you can conceptually think of as how hard it is working, will fall to zero as it runs out of steam.

Now that we have a picture of what is going to happen we can do the math. Get the differential equation by doing our modified Kirchhoff’s loop rule (actually Faraday’s Law) around the loop:

$$-RI - L\frac{dI}{dt} = 0 \rightarrow \frac{dI}{dt} = -\frac{R}{L} I$$  \hspace{1cm} (2)

The solution to this differential equation is an exponential decaying current: $I = I_0 e^{-t/\tau}, \tau = L/R$

The voltmeters across the resistor and inductor read: $V_R = IR = I_0 R e^{-t/\tau}$ & $V_L = -L\frac{dI}{dt} = \frac{LI_0}{\tau} e^{-t/\tau}$  \hspace{1cm} (3)

A plot of $I$, $V_R$ or $V_L$ looks like the figure at left, with the value decaying exponentially from its initial value to zero after a long time.