8.02                  Spring 2004

Experiment 10: Undriven LRC Circuits

OBJECTIVES

1. To determine the inductance $L$ of a coil, both with and without an iron core, on the AC/DC Electronics Lab circuit board.
2. To determine the quality factor of an LRC circuit.
3. To observe electrical oscillations, measure their frequencies, and verify energy relationships in an LRC circuit.

INTRODUCTION

Free Oscillations in LC and RLC circuits

(For purposes of clarity, these experiment instructions will use lower-case letters, $q(t)$ and $i(t)$ to denote time-varying circuit quantities.)

Consider a series RLC circuit shown in Figure 15.1.

![RLC Circuit](image)

Figure 15.1 RLC Circuit with external voltage removed

Applying the Kirchhoff’s voltage rule, the circuit equation for the RLC circuit without any external voltage is

$$0 = L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C}. \quad (10.1)$$
In this experiment, the internal resistance of the inductor is non-negligible. The sum of the inductor resistance $R_L$ and any added series resistance $R$ will be added to obtain the total resistance $R_T = R + R_L$.

For the situation where the capacitance is not part of the circuit, the current in an $LR$ circuit is derived in the 8.02 Course Notes, Section 11.4, and given in Equations (11.4.7) and (11.4.13):

$$i(t) = \frac{E}{R_T} \left(1 - e^{-t/\tau} \right)$$

for an increasing current and

$$i(t) = \frac{E}{R_T} e^{-t/\tau},$$

for a decreasing current, where the time constant is $\tau = L / R_T$.

For a circuit with capacitance, with appropriately small resistance (“underdamped”), the current can be represented as

$$i(t) = I_0 e^{-\gamma t} \cos \omega t$$

where $\gamma = R_T / 2L$, $\omega^2 = \frac{1}{LC} - \frac{R_T^2}{4L^2}$ (see the 8.02 Course Notes, Section 11.6 for a derivation). A plot of $i(t)$ for a typical underdamped circuit is shown in Figure 1.

![Figure 1 Exponential decay of current oscillations in a typical RLC circuit](image)
Since the coefficient of exponential decay, $\gamma = R_L / 2L$, is proportional to the resistance, we see that the current will fall off more rapidly as the resistance increases.

**Quality factor of the RLC circuit**

We define the quality factor $Q$ (not to be mistaken for a charge) of an RLC circuit is most reliably defined in terms of energy or power considerations; for a circuit with low damping ($\omega = \omega_0$), we can define the quality factor (see the 8.02 Course Notes, Equation (11.8.7) or Equation (12.4.6)) as

$$Q = \frac{\omega_0 L}{R_L} = \frac{\omega_0}{2\gamma}.$$

(10.2)

**Energy Relationships in RLC circuits**

As the current oscillate in such circuits, energy may be stored in both the magnetic field of the inductor

$$U_B = \frac{1}{2} Li^2$$

(10.3)

and in the electric field of the capacitor

$$U_E = \frac{1}{2} CV^2 = \frac{1}{2} q^2$$

(10.4)

The energy stored in the electric and magnetic fields is simply the sum:

$$U = \frac{1}{2} Li^2 + \frac{1}{2} CV^2$$

(10.5)

However, this energy is gradually being lost as heat in the resistor at the rate $I^2R_L$:

$$\Delta U = -(U(t) - U(t = 0)) = \int_0^t i^2 R_L \, dt$$

(10.6)

From this expression it can be seen that for low damping the quality factor is proportional to the total energy divided by the energy dissipated in one oscillation.

**EXPERIMENTAL SETUP**
A. Computer

If it is not done already, connect the *Science Workshop 750 Interface* to the computer using the SCSI cable. Connect the power supply to the 750 Interface and turn on the interface power. Always turn on the interface before powering up the computer. Turn on your computer.

B. AC/DC Electronics Lab circuit board

Connect the black banana plug cord from the OUTPUT ground port of the 750 Interface to the banana jack located in the lower right corner on the AC/DC Electronics Lab circuit board. Connect the red banana jack with alligator clip to the positive OUTPUT port of the 750 Interface. During the course of the exercises below, you will connect the alligator clip to various places in order to close the circuit. You will also connect the Voltage Sensor to measure the inductor voltage. For this experiment, in which all of the circuit elements are in series, you will be able to measure and record the Output Current from the 750, so the Current Sensor is not part of this experiment.

MEASUREMENTS

Part 1. Resistance and Inductance of the Coil

Resistance:

*Circuit Diagram:* Connect, using the red alligator clip, the positive OUTPUT port to the right side of your coil. Using a wire, connect the left side of your coil to the banana jack that is connected to the OUTPUT ground port (black) of the 750 Interface (see Figure 2).

![Figure 2 Circuit diagram for measuring the resistance of the coil](image)

*DataStudio File:* Download and open the *Data Studio file exp10.ds*. If the following settings have not been made, you will have to change them.

- The *Signal Generator* should be set to a *Positive Square Wave* with a frequency of 20 Hz and amplitude of 1 volt. The *Sample Rate* should be 1000 Hz.

- A graph has been set up to display the Output Current and Output Voltage.
Click Start. Use the Smart Tool (sixth icon from the left) to measure the current and then compute the resistance $R_L$ of the inductor. Check your result using the multimeter (recall that you need to make sure that the multimeter is zeroed before you take a measurement).

**Question 1:** Measure and record the resistance $R_L$ of your coil.

**Inductance (with and without an iron core)**

Connect the 10-Ω resistor in series with the inductor, as shown in Figure 3.

![Circuit diagram for measuring the inductance of the coil](image)

**Figure 3** Circuit diagram for measuring the inductance of the coil

In the DataStudio window, click Start. The Output Current graph displays the familiar behavior of an $LR$ circuit (see Figure 4).

![Exponential decay of current in LR circuit](image)

**Figure 4** Exponential decay of current in $LR$ circuit

**DATA ANALYSIS**

When the Positive Square Wave voltage switches to 0 volts, the total resistance is $R_T = R + R_L$. The current in the circuit decays exponentially and is given by
\[ i(t) = i_0 e^{-\frac{R}{L}t}, \]  
\( (10.7) \)

where \( i_0 = \mathcal{E}/R \) is the current in the inductor at the time when the voltage drops to zero.

Taking the natural logarithm of both sides of the above equation gives

\[ \ln(i) = -\left(\frac{R}{L}\right)t + \ln(i_0), \]  
\( (10.8) \)

which means that a graph of \( \ln(i) \) vs. \( t \) has a slope equal to \( -\left(\frac{R}{L}\right) \).

The following instructions are reproduced from “Data Studio Instructions”, and may have been used to find the time constant of an \( RC \) circuit in a previous experiment.

Use the Zoom Select tool (fourth icon from the left on the menu bar for the Graph window) to draw a box around data points in the exponentially decaying region of the Output Current and then choose Copy from the Edit menu. Click the Calculator button on the menu bar. In the Calculator window click New. The variable \( x \) should be highlighted in the Definition window. Click on the Scientific button and scroll down and click on \( \ln(x) \). Scroll down on the Variables menu and click on Data Measurement. In the Please Choose a Data Source window, scroll and click on Current, ChA and click OK. Then click the Accept button in the Calculator window. A calculator icon with your equation should appear in the Data window. Drag that calculator icon to the Graph icon in the Display window. A fairly complicated graph will appear, due to the fact that many of the measured currents are close to zero. Use the Zoom Select to isolate data where the function is linear. You might see fluctuations in the data due to approximations associated with the sampling rate. You can use Zoom Select to choose the region where there are the smallest fluctuations. Use the mouse to highlight a region of data. Once you have isolated this region, click on the Fit button, scroll down and click to Linear Fit. You will obtain the \( y \)—intercept of the best-fit line, which corresponds to the maximum current \( i_0 \), and the slope, which can be used to calculate the measured time constant.

**Question 2:** Calculate the value of the inductance with and without the core and record your results on the tearoff sheet.

**Part 2: Free Oscillations of the \( RLC \) Circuit**

**Circuit Diagram:** Now put a \( C = 10 \mu F \) capacitor in series with the coil (without its core) and the \( R = 10 \Omega \) resistor (as shown in Figure 5).
Figure 5 Circuit diagram for the RLC circuit

Repeat the above procedure to record the current and voltages for this RLC circuit. You may want to delete previous data runs which may overlap the graphs of this data run.

**Question 3:** Determine the period $T$ of these oscillations by using the Smart Tool to measure the time interval between when the current is zero. The period is twice the time interval between successive zeroes. Calculate the frequency, $f = 1/T$, of these oscillations and record your results on the tearoff sheet.

**Question 4:** For small values of resistance, the oscillation frequency is approximately $f = 1/(2\pi\sqrt{L/C})$. For your circuit parameters, compute the expected value of $f$ and compare it to your measured value. Do you expect your result to be greater, equal, or less than the measured value? Answer on the tearoff sheet.

**Question 5:** What is the quality of this circuit? (Refer to Equation (10.2) above). Give your prediction on the tearoff sheet.

**Question 6:** Using your plot of current vs. time, measure $n$, the number of cycles the current oscillates until the exponential envelope of the current falls off by a factor of $e^{-1}$. This may take some inventiveness on the experimenters’ part, since there is no way to insure the maxima as depicted in Figure 15.2 correspond to the maxima of $\cos \omega t$. It’s a math thing. How does your measurement agree with your prediction? Can you explain any discrepancies?

**Part 3: Observe the Energy in the RLC Circuit**

**Circuit Diagram:** In order to investigate the energy in an RLC circuit, you will need to use a circuit consisting of just an inductor and capacitor. The resistance of the inductor, $R$, reminds us that this is a typical RLC circuit. Insert a Voltage Sensor into Channel B on the 750 Interface and connect it across the terminals of the capacitor (see Figure 6).
The Data Studio file exp10.ds should have a window in which the predicted energies in the capacitor and inductor, and the total electromagnetic energy, are displayed.

[The calculated energies assume the values $L = 8\, \text{mH}$ and $C = 10\, \mu\text{F}$, so if your circuit parameters are very different you should change these numbers in the calculator window.]

Click Start to record data. The oscillations you previously observed should appear in the same Graph 1 window. In Graph 3, the energies are plotted. Because the energies are very small, they have been multiplied by a million, that is, the units are micro-joules. You will want to manually expand the scale and move the data around within the Graph 3 window in order to make it more visible. Figure 7 shows an example of what you should expect to find.

**Figure 6** Circuit diagram for investigating the energy relationship in an $RLC$ circuit

**Figure 7** Energy in an $RLC$ circuit

**Question 8:** The circuit is losing energy most rapidly at times when the graph of total energy is steepest; these times occur at about the same times that the magnetic energy reaches a local maximum. Briefly explain why.
Experiment Summary 10: Undriven \( RLC \) Circuit

Group and Section ___________________________ (e.g. 10A, L02: Please Fill Out)

Names ________________________________

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MEASUREMENTS

Part 1. Resistance and Inductance of the Coil

Question 1: Measure and record the resistance \( R_L \) of your coil.

Coil Resistance: \( R_L = \) ___ \( \Omega \)

Question 2: Record the inductance of your coil, with and without the iron core:

Coil inductance without core: \( L = \) ___ H

Coil inductance with core: \( L = \) ___ H

Part 2: Free Oscillations of the \( RLC \) circuit

Question 3: Determine the period \( T \) of the oscillations of the \( LRC \) circuit. The period is twice the time interval between successive zeroes. Calculate the frequency, \( f = 1/T \), of these oscillations and record your results here.

Answer: \( f = \) ___ Hz
**Question 4:** For small values of resistance, the oscillation frequency is approximately \( f = \frac{1}{2\pi\sqrt{LC}} \). For your circuit parameters, compute the expected value of \( f_{\text{predicted}} \) and compare it to your measured value. Do you expect your result to be greater, equal, or less than the measured value?

**Answer:** \( f_{\text{predicted}} = \) ___ Hz

**Question 5:** What is the quality of this circuit?

**Answer:** \( Q = \) ___ (trick question; what are the units of \( Q \)?)

**Question 6:** Using your plot of current vs. time, measure the number of cycles the current oscillates until the exponential envelope of the current falls off by a factor of \( e^{-1} \). How does your measurement agree with your prediction? Can you explain any discrepancies?

**Answer:** \( n = \) number of cycles = ___ (another trick question; what are the units of \( n \)?)

**Question 7:** Describe the qualitative difference between the two graphs. Calculate the frequency of the oscillations for the circuit without the external resistance. Now measure the frequency. Does your measured value of the frequency correspond to your theoretical prediction?

**Answer:**

**Part 3: Observe the Energy in the RLC Circuit**

**Question 8:** The circuit is losing energy most rapidly at times when the graph of total energy is steepest; these times occur at about the same times that the magnetic energy reaches a local maximum. Briefly explain why.

**Answer:**