Preparation for Experiment 4, Part 1

A rotor of moment of inertia $I_R$ and outer radius $R$ spins about a vertical axis. The rotor mount is not friction free. Rather, there is a small constant friction torque of magnitude $\tau_f$. A string of negligible mass is wound around the outside of the rotor. The string is attached to the rotor in a way that allows it to come free when the end is reached. The other end of the string is attached to a weight of mass $m$ hanging from a massless frictionless pulley. When the system is released from rest the angular velocity of the rotor increases linearly at a rate $\alpha_1$ under the influence of the tension in the string and the torque due to friction. After the string detaches from the rotor, the rotor’s angular acceleration becomes $\alpha_2$ (a negative quantity) due to the friction torque. A qualitative graph of $\omega(t)$ is shown below.

a) Find expressions for $I_0$ and $\tau_f$ in terms of some or all of the quantities $\alpha_1$, $\alpha_2$, $m$, $R$, and $g$ the acceleration of gravity.
Now imagine that the rotor is spinning down freely and at \( t = 0 \), when its angular velocity is \( \omega_1 \), a washer of moment of inertia \( I_W \) is dropped onto it. After a time \( \Delta t \) the washer comes to rest on the rotor. At that instant they have the common angular velocity \( \omega_2 \).

\[
\begin{align*}
\text{not rotating} & \quad I_W \quad \omega_1 \quad \text{rotating with rotor} \\
\end{align*}
\]

b) Find the moment of inertia of a uniform ring of outer radius \( R_o \), inner radius \( R_i \) and total mass \( M \). You may find it useful to recall that \((R_o^4 - R_i^4) = (R_o^2 + R_i^2)(R_o^2 - R_i^2)\)

c) If \( \Delta t \) is so short that no angular momentum is lost to the friction torque, what is \( \omega_2 \) in terms of \( I_R, I_W \) and \( \omega_1 \)?

d) Now assume that \( \Delta t \) is long enough that the friction torque \( \tau_f \) has to be taken into account. What is \( \omega_2 \) in terms of \( I_R, I_W, \Delta t, \tau_f \) and \( \omega_1 \)?

You will make use of all of the expressions found above in performing the experiment. You can check to see if your expressions are correct by comparing them with the corresponding expressions given in the Lab Writeup.

**Preparation for Experiment 4, Part 2**

Experiment 4 follows rather closely the situations described in Part 1. Four data files similar to those you will take in class can be found at http://web.mit.edu/8.01t/www/Prelab\%20Data/. There is one data file for each phase of the experiment. In this part of the PreLab you will analyze this data and fill out a Results Page identical to the one you will have in class.

**MOMENT OF INERTIA**

Open the file AMmassFall.txt in an Excel or OpenOffice worksheet. The two columns represent the time and the angular velocity for the experiment shown in the top figure on page 1 of this PreLab. Plot this data in a separate sheet. Use the XY(scatter) format without connecting lines. Minimize the size of the points used to represent the data on the chart. You should see a noisy version of the bottom figure on page 1. Pick a range of times that best represents the linear rise of \( \omega \) with time. Try to avoid the regions of transitional behavior near the bottom and the top of the curve. Go back to the original worksheet and select the cells containing the times and corresponding angular velocities in this region. Plot this region of the data in a new sheet. Use Trendline to fit this region of the data to a linear curve, thus determining the best fit value for \( \alpha_1 \). Enter this value on the Results Page. Find and enter \( \alpha_2 \) in a similar manner. For future reference, estimate the average value of \( \omega \) on this plot and enter it on the Results Page.
The mass of the weight is 55 grams. The radius of the rotor about which the string is wrapped is 1.27 cm. Use the expressions you derived in a) to find the moment of inertia of the rotor $I_R$ and the friction torque $\tau_f$. Enter these values on the Results Page.

The angular velocities attained in the weight drop experiment are lower than the velocities that are involved in the subsequent experiments. We have found that the friction torque is not a constant. Rather, it tends to increase at higher speeds. The next experiment is designed to determine the friction torque as a function of angular velocity.

**VARIATION OF FRICTION TORQUE**

Open the file AMSpinDown.txt. The two columns give the time and angular velocity as the rotor slows down from a high rotation rate in the absence of any external torque except that due to friction in the bearing. Make a plot of angular velocity verses time on a new sheet. If the friction torque were constant, the slope of the curve would be constant. Inspection of the plot, perhaps with the aid of the straight edge of a piece of paper, shows that this is not the case. The slope is higher at higher angular velocities. We will make a quantitative determination of the angular deceleration as a function of angular velocity.

Use Trendline to make a second order (through $X^2$) fit to the data. You should find that the resulting curve describes the data for $\omega$ verses $t$ quite well. Take the derivative of the expression given by the fitting routine (you did remember to check display equation on chart didn’t you) to find a model for $\alpha$ as a function of time. However, we want to know $\alpha$ as a function of $\omega$.

Here is a relatively simple way to find an analytic model for $\alpha$ as a function of $\omega$. Go back to the data sheet. Time is in column A and the measured Omega is in column B. Skip a column for neatness then in column D make a heading OmegaM and in column E a heading AlphaM. The M stands for model. Using the model expressions for $\omega(t)$ and $\alpha(t)$ you got from the fitting routine, and the values of $t$ from column A, fill out the cells in columns D and E. Now select the contents of columns D and E (starting with the heading row and selecting down to the end of the filled cells) and make a plot of $\alpha$ verses $\omega$ in a separate sheet. Note that there is no noise, since you are plotting analytic results. Use Trendline to fit this data to second order. The result will be an analytic expression for $\alpha(\omega)$.

OpenOffice does not offer a polynomial fit, though you will have that capability in class on Excel. So I am giving you here the results of my fits. Use these to continue your PreLab analysis.

Fit to $\omega$ verses $t$: $y = 0.6378 \times 10^2 - 40.02 x + 580.7$
Fit to $\alpha$ verses $\omega$: $y = 2.08 \times 10^{-5} x^2 - 5.48 \times 10^{-2} x - 15.19$
Now do a consistency check. Take the average value of $\omega$ you found when determining $\alpha_2$ as part of the $I_R$ measurement. Plug that value into the expression you just found and enter the result on the Results Page. How does the model $\alpha$ compare with the measured one? I found that the magnitude of the model result was only about 75% of the measured value. I would have expected a better agreement. Perhaps the friction is due in part to the oil or grease in the bearings, the bearings heat up at higher speeds, and the viscosity of the oil or grease goes down. It would take a while for the bearings to heat up during repeated high speed use, but the moment of inertia determination was done first, while the bearings were cooler. But this is just a guess on my part. In class we will see how this fraction varies from apparatus to apparatus.

In the next experiments we will be changing the moment of inertia. Thus we will need an expression for $\tau_f$. Multiply your expression for $\alpha$ by the moment of inertia you found earlier and enter this expression $\tau_f(\omega)$ on the Results Page.

**FAST INELASTIC COLLISION**

For the data supplied with this PreLab, the dropped washer had a mass of 79.3 grams and inner and outer radii 1.35 and 3.18 cm. Compute $I_W$ and enter it on the Results Page.

Open the file AMfastCollide.txt. The two columns give the time and angular velocity during a “Fast” inelastic collision. Plot the data on a separate sheet. Determine the time $t_1$ when the collision appears to begin and the time $t_2$ when the collision appears to end. Enter these times on the Results Page.

Go back to the data sheet and select the data up to $t_1$. Plot this range of data on a separate sheet. Use Trendline to do a linear fit to the data. From that fit, determine the value of the angular velocity $\omega_1$ at $t_1$. Enter that value on the Results page.

Go back to the data sheet and select the data from $t_2$ to the end. Plot this range of data on a separate sheet. Use Trendline to do a linear fit to the data. From that fit, determine the value of the angular velocity $\omega_2$ at $t_2$. Enter that value on the Results page. Compute the average of $\omega_1$ and $\omega_2$ and add this to the Results Page.

The expression you found for $\omega_2$ in d) can be split into a part that would be expected if there were no friction torque, and a correction due to friction.

$$\omega_2 = \frac{I_R \omega_1 - \tau_f \Delta t}{I_R + I_W}$$

$$= \frac{I_R}{I_R + I_W} \omega_1 - \frac{\tau_f \Delta t}{I_R + I_W}$$

$$= \omega_{\text{no friction}} + \text{correction}$$
Use your expression for $\tau_f(\omega)$ together with the average $\omega$ during the collision to find the value of $\tau_f$ to use in this expression. Enter it on the Results Page. Calculate the two contributions to the predicted $\omega_2$ and enter them on the Results Page. Also enter the resultant sum. You should find that the predicted value differs from the measured value by a few percent. You should also see that the friction correction is not negligible on the scale of the observed difference.

**SLOW INELASTIC COLLISION**
Open the file AMslowCollide.txt. Carry out exactly the same analysis you did for the fast collision. Keep in mind, however, that the stretching out of the collision time has nothing to do with the friction in the bearing. It is due to the time necessary for the washer and rotor to reach a common angular velocity. This process is carried out by forces internal to the system under consideration, thus it does not change the angular momentum of the system. The external torque, that due to the friction in the bearing, simply has a longer time to act on the system than it did in the fast collision; thus it will produce a larger percentage correction to the result one would expect if the angular momentum of the system were conserved. You will see this as you fill out the relevant parts of the Results Page for the slow collision.

Print out the charts you used for curve fitting. Bring them to class and hand them in along with your in-class experimental write up. Add a statement to the charts you hand in saying that you did the work on your own, then sign it.

No credit will be given for the experiment without these charts.
Results Page for Experiment 4

**MOMENT OF INERTIA**

\[ \alpha_1 \]  

\[ \alpha_2 \]  Average \( \omega \) while finding \( \alpha_2 \)  

\[ I_R \]  

\[ \tau_f \]  Model prediction for \( \tau_f \)

**VARIATION OF FRICTION TORQUE**

Model expression for \( \tau_f(\omega) \)

**FAST COLLISION**

\[ t_1 \]  

\[ \omega_1 \]  \[ \frac{\omega_1 + \omega_2}{2} \]  

\[ t_2 \]  

\[ \omega_2 \]  

\[ \tau_f \]  

\[ \omega_2 \text{ predicted} \]  

\[ \omega_2, \text{ no } \tau_f \]  

\[ \tau_f \text{ correction} \]  

\[ \omega_2 \text{ predicted} \]

**SLOW COLLISION**

\[ t_1 \]  

\[ \omega_1 \]  \[ \frac{\omega_1 + \omega_2}{2} \]  

\[ t_2 \]  

\[ \omega_2 \]  

\[ \tau_f \]  

\[ \omega_2 \text{ predicted} \]  

\[ \omega_2, \text{ no } \tau_f \]  

\[ \tau_f \text{ correction} \]  

\[ \omega_2 \text{ predicted} \]