Purpose of the Experiment:
In this experiment you investigate rotational collisions and the conservation of angular momentum in rigid body rotational dynamics. It is the rotary counterpart of linear collisions.

The heart of the experiment is a high quality DC motor to spin a rotor up to several hundred radians per second. When power to the motor is shut off, it serves as a tachometer-generator whose output voltage is proportional to the angular velocity of the rotor; thus the angular velocity of the rotor can be determined by measuring the output voltage. When you hold down the red pushbutton switch on the apparatus, power is applied to the motor; when you release it, the rotor coasts and the output voltage the motor generates can be read by the computer. This experiment will give you experience in

- measuring and calculating moments of inertia,
- measuring and calculating the effects of external torques, including torque due to friction, and
- determining the changes in angular momentum during inelastic rotational collisions.

Setting Up the Experiment:
Plug the rotary motion apparatus into its power supply; you should see the LED in the plastic pipe elbow come on.
The experiment is longer than any we have done before and has two parts; extra time has been allotted to do it. First, you will calibrate the equipment and measure the moment of inertia of the rotor. Second, you will use the calibrated apparatus to study rotational collisions.  

At the start of the experiment, use two voltage sensor plugs to connect the output from the apparatus to inputs A and B on the SW750 interface box. Connect the generator output voltage (the jacks farthest from the power input connector) to input A and the phototransistor output to input B.  

The data acquisition and analysis for both experiments will be done with the LabVIEW program called *AngularMomentum*.  

The program operation is controlled by the main pull-down menu at the upper left of the graph and the three tabs labeled “Graph”, “Run and Fit Parameters” and “Calibrate” on the front panel. The Calibrate tab is an addition to what you have become used to our previous LabVIEW programs. It is there because two calibrations must be done in this experiment. Before you can carry out any rotational collision measurements, you must first calibrate the tachometer-generator and then you must measure the moment of inertia of the rotor. These are the tasks of Part I; in Part II you will study rotational collisions.  

The first task is to calibrate the tachometer generator output, i.e., what is $\omega$ for 1 V output? When the Calibrate tab is active (that means it’s the one whose contents you can see) and the “Which Calibration?” pull-down menu on the tab is set to Tachometer, the program will measure the voltages on channels A and B and assume that data files saved by or read into the program contain two voltages as a function of time. Also, the Fit Data option of the main pull-down menu will use these two voltages to calibrate the generator, in a way that you will see presently.  

Otherwise—when the Calibrate tab is not active (visible) or it is active with the pull-down menu on the tab set to Moment of Inertia—the program measures only the voltage on input channel A and uses the generator calibration result to convert the voltage to angular velocity. Data files saved by the program will contain only angular velocity as a function of time.  

**Cursor Control:**  
This widget above the graph helps you position the cursor more accurately. The button in the middle lets you select which cursor moves. The arrows move the cursor one data point each time you click them. (On Windows machines the keyboard left and right arrow keys do the same thing.) You can still position the cursors by dragging, but this may be easier for precise movement.
Part I: Calibration

Calibrating the Generator:
Check to make sure there is a black sticker on the white plastic centerpiece of the rotor where it will be illuminated by the LED; the reflected light will be detected by the phototransistor. (The voltage output will be higher when the light is reflected from the black tape and lower when more light is reflected from the white plastic.)

Set the voltage sample rate to 2000 Hz from the pull-down menu and type a run time of 0.25 (s) into the window. For this calibration you want to measure both the phototransistor output voltage and the voltage from the generator. Then perform the following steps.

1. Click the Calibrate tab and make sure the “Which Calibration?” pull-down menu is set to Tachometer.

2. While the Calibrate tab is still active (visible) choose Measure from the main pull-down menu; the START button will become brighter green, indicating the program is ready to make the measurement.

3. Spin the motor up for several seconds, release the red button and allow the motor to coast for about a second, then click the START button (or type the Esc key on the keyboard). The START button will briefly turn red and say STOP while the computer makes the measurement.

After the measurement is over, click the Graph tab and you will have a plot of the two voltages, as in the figure below. The purple curve (Plot 0) is the generator output voltage and the green one (Plot 1) is the phototransistor voltage.
To calibrate the generator, you need to find the time between the first and last fully visible phototransistor peaks on the graph. Here is how to do that.

1. Place one cursor at the center of the first complete voltage peak and the other at the center of the last complete voltage peak of the phototransistor output. You can do this roughly by dragging the the cursor onto the peak. Then use the zoom control (center button on the graph palette) to expand each peak in turn and position each cursor more accurately in the center of the peak.

2. Next, count the number of rotation periods between the two peaks; then click the Calibrate tab. You can type in the likely errors in the voltage (Sigma V) and time (Sigma T) measurements, but the default $\sigma_V = 0.01 \text{ V}$ and $\sigma_T = 0.0005 \text{ s}$ are OK.

3. Make sure the pull-down menu on the tab says Tachometer, and choose Fit Data from the main pull-down menu. A dialog window will open so you can type in the number of rotation periods. As soon as you enter the number of periods, the program will calculate the average generator voltage and the average angular velocity for the time between the cursors and fill in the remaining fields in the box. The results will be saved in the computer’s memory.

All of the fields in the Tachometer calibration box should now have numbers in them:

That completes the tachometer calibration.

Note: it’s a good idea to write down the tachometer calibration (Rad/Sec/Volt) and its standard deviation. Then if the computer crashes, or some other accident occurs, you can recover by typing these numbers into the fields on the Calibrate tab and the program will use them.
Moment of Inertia Calibration

The second calibration you must do is to measure the moment of inertia of the rotating parts of the apparatus. To measure the rotor moment of inertia, a known torque is applied by a falling weight and the angular acceleration is measured. Use a 50 gm brass weight with a 5 gm plastic holder. The apparatus should be set up near the edge of the table, as in the photograph below.

Setting up the string and weight: Take the knot tied in one end of the string and place it in the kerf cut into the brass washer on the rotor. The string should wrap around the rotor several times, pass over the pulley and be fastened to the weight as shown in the photo. The length of string was chosen so that the string will completely unwind and pull out of the kerf just before the weight hits the floor.

Measuring the moment of inertia: Choose “Moment of Inertia” from the “Which Calibration?” pull-down menu on the Calibrate tab. Set the voltage sample rate to 500 Hz and the run time to 4.0 s. Choose Measure from the main pull-down menu and the START button should light up green. Wind up the string around the rotor so the weight is near the pulley and hold the rotor so it cannot unwind. Click the START button (Esc key) and release the rotor a fraction of a second later. After the 4 seconds have elapsed, click the Graph tab and you should see a graph like the one at the top of the next page. Save the data to an Excel file. Label it AMmassFall.
Theory of the analysis: While the weight is falling, the rotor angular velocity $\omega$ increases ($\alpha > 0$) at a constant rate. After the string pulls out, $\omega$ decreases ($\alpha < 0$) at a constant rate until the rotor stops. The decrease is caused by friction. As the accelerations (slopes of $\omega$ vs. $t$) are constant, the torques must be constant.

Let’s call $\alpha_1$ (a positive number) the angular acceleration while the weight is falling and $\alpha_2$ (a negative number) the angular acceleration while the rotor is coasting to a stop. Let $\tau_f$ be the magnitude of the torque due to friction. During the coasting period, we must have

$$-I_R \alpha_2 = \tau_f$$

where $I_R$ is the moment of inertia of the rotor.

The equations of motion while the weight is falling are a bit more complicated. Suppose $T$ is the tension on the string. The white plastic part of the rotor has a diameter of 1.00 in or a radius of $r = 12.7$ mm. The string will apply a torque to the rotor given by

$$\tau_s = rT.$$ 

A free body diagram of the falling weight will give

$$T = mg - ma$$

where $m$ is the mass of the weight and $a$ is the magnitude of the vertical acceleration of the weight.
Kinematics tells us that

\[ a = r\alpha_1. \]

The final equation we need is (the friction opposes \( \tau_s \))

\[ I_R \alpha_1 = \tau_s - \tau_f. \]

After eliminating \( \tau_f, \tau_s, T \) and \( a \) from these equations, the result is (remember, \( \alpha_2 < 0 \))

\[ I_R = \frac{mr(g - r\alpha_1)}{\alpha_1 - \alpha_2}. \]

**Doing the analysis:** Open the file AMmassFall in Excel. The two columns represent the time and the angular velocity. Plot this data in a separate sheet. Use the XY(scatter) format without connecting lines. Minimize the size of the points used to represent the data on the chart. You should see a graph similar to the LabVIEW version on the previous page. Pick a range of times that best represents the linear rise of \( \omega \) with time. Try to avoid the regions of transitional behavior near the bottom and the top of the curve. Go back to the original sheet and select the cells containing the times and corresponding angular velocities in this region. Plot this region of the data in a new sheet. Use Trendline to fit this region of the data to a linear curve, thus determining the best fit value for \( \alpha_1 \). Enter this value on the Results Page. Find and enter \( \alpha_2 \) in a similar manner. For future reference, estimate the average value of \( \omega \) on this plot and enter it on the Results Page.

The mass of the weight is 55 grams, The radius of the rotor about which the string is wrapped is 1.27 cm. Use the expressions above to find the moment of inertia of the rotor \( I_R \) and the friction torque \( \tau_f \). Enter these values on the Results Page.

**Measuring the Variation in Friction Torque:** Actually, the friction torque acting on the rotor varies slightly with \( \omega \). The variation is not enough to matter for the calibration you have just completed, but it will be significant when you analyze the rotary collisions between washers, where \( \omega \) can be much larger than 100 s\(^{-1}\), so it should be measured. This measurement is straightforward. Choose a sample rate of 200 Hz and a run time of 10 s. Select Measure from the main pull-down menu, run the motor up close to its maximum speed, then start it coasting and click START (or the Esc key) at about the same time. You should get a graph like the one on the next page.

You can see this plot is not linear, that is, \( \alpha = -\tau_f/I_R \) depends slightly on \( \omega \) since the slope is higher at higher angular velocities. Save these data to an Excel file and label it AMSpinDown.
Open the file AMSpinDown. The two columns give the time and angular velocity. Make a plot of angular velocity verses time on a new sheet. It should look like the LabVIEW version above. We can now make a quantitative determination of the angular deceleration as a function of angular velocity.

Use Trendline to make a second order (through $X^2$) fit to the data. You should find that the resulting curve describes the data for $\omega$ verses $t$ quite well. Take the derivative of the expression given by the fitting routine to find a model for $\alpha$ as a function of time. However, we want to know $\alpha$ as a function of $\omega$.

Here is a relatively simple way to find an analytic model for $\alpha$ as a function of $\omega$. Go back to the data sheet. Time is in column A and the measured Omega is in column B. Skip a column for neatness then in column D make a heading OmegaM and in column E a heading AlphaM. The M stands for model. Using the model expressions for $\omega(t)$ and $\alpha(t)$ you got from the fitting routine, and the values of $t$ from column A, fill out the cells in columns D and E. Now select the contents of columns D and E (starting with the heading row and selecting down to the end of the filled cells) and make a plot of $\alpha$ verses $\omega$ in a separate sheet. Note that there is no noise, since you are plotting analytic results. Use Trendline to fit this data to second order. The result will be an analytic expression for $\alpha(\omega)$.

In the next experiments we will be changing the moment of inertia. Thus we will need an expression for $\tau_f$. Multiply your expression for $\alpha$ by the moment of inertia you found earlier and enter this expression $\tau_f(\omega)$ on the Results Page.

Now do a consistency check. Take the average value of $\omega$ you found when determining $\alpha_2$ as part of the $I_R$ measurement. Plug that value into the expression you just found and enter the result on the Results Page. How does the model $\tau_f$ compare with the measured one? Calculate the ratio of the model value of $\tau_f$ to the measured one. Report this value to the class TA who will enter it on the board. We would like to see how this fraction varies from apparatus to apparatus.
Part II: Angular Collisions

Follow the same procedures as in Part I (pages 1 and 2), with the following differences.

- Connect only the generator output (jacks farthest from the power input connector) to input A of the SW750.

- Click the Calibrate tab and, only if necessary, type your calibration results from Part I (Rad/Sec per Volt and its standard deviation) into the appropriate fields in the Tachometer Calibration box.

- Find the mass and the inner and outer radii of the washer you will be using. Calculate the moment of inertia of the washer, $I_W$, and enter it on the Results Page.

- To study the collisions, set the Sample Rate to 200 Hz and the Run Time to 4.00 seconds.

Hints for Successful Collisions:

You will get better results when you drop the washer onto the spinning rotor in the best way. Here is what I found helps.

- Hold the washer level with the hole in the washer centered above the rotor axis and just above the top of the rotor before you drop it. I suggest a separation of about 1/4 inch or 1/2 centimeter.

- Release the washer so it stays level as it falls. This is easier to do if someone else operates the computer.

Velcro:

If you look carefully at the Velcro, you will see it comes in two kinds. One has little hooks and the other is softer and looks more furry. The two kinds stick together when the hooks latch into the “fur.”

The top of the washer that is a permanent part of the rotor has fur. One of the washers you can drop has hooks on one side and fur on the other. The other washer you can drop has hooks on one side and no Velcro on the other side. This makes three kinds of collisions possible. If you drop hooks onto fur the washers stick almost instantly and you get a fast collision. If you drop brass or fur onto fur, the washers slide a while before reaching the same $\omega$ and a slow collision (lasting a few 100 ms) results.

First you should investigate a slow collision, as it is easier to understand the detailed behavior of it than it is for a fast collision.
A Slow Inelastic Collision:

Try dropping a washer to produce both brass-on-fur and a fur-on-fur collisions. For each collision you should get a graph something like this one.

Here are the criteria to use in deciding if you have reasonably good data:

- There is a fairly clear break in slope at the beginning and the end of the collision, making it easier to determine the duration of the collision.

- The slope of $\omega(t)$ is fairly constant during the collision and there is little sign of wobbling of the washer as it falls onto the rotor.

If a brass-on-fur collision seems reasonable, save the data to an Excel file labeled AMslowCollideBF.

Repeat for a fur-on-fur collision, and save what seems to be a reasonable collision to an Excel file labeled AMslowCollideFF. Then decide which of the slow collisions you want to analyze and begin the analysis. The graph above is a fur-on-fur collision.
Analysis of the Slow Collision:

Open the Excel file for the collision you have chosen. The two columns give the time and angular velocity. Plot the data on a separate sheet. Determine the time $t_1$ when the collision appears to begin and the time $t_2$ when the collision appears to end. Enter these times on the Results Page.

Go back to the data sheet and select the data up to $t_1$. Plot this range of data on a separate sheet. Use Trendline to do a linear fit to the data. From that fit, determine the value of the angular velocity $\omega_1$ at $t_1$. Enter that value on the Results page.

Go back to the data sheet and select the data from $t_2$ to the end. Plot this range of data on a separate sheet. Use Trendline to do a linear fit to the data. From that fit, determine the value of the angular velocity $\omega_2$ at $t_2$. Enter that value on the Results page. Compute the average of $\omega_1$ and $\omega_2$ and add this to the Results Page.

The expression you found in the PreLab for $\omega_2$ can be split into a part that would be expected if there were no friction torque, and a correction due to friction.

\[
\omega_2 = \frac{I_R \omega_1 - \tau_f \Delta t}{I_R + I_W}
\]

\[
= \frac{I_R}{I_R + I_W} \omega_1 - \frac{\tau_f \Delta t}{I_R + I_W}
\]

\[
= \omega_{\text{no friction}} + \text{correction}
\]

Use your expression for $\tau_f(\omega)$ together with the average $\omega$ during the collision to find the value of $\tau_f$ to use in this expression. Enter it on the Results Page. Calculate the two contributions to the predicted $\omega_2$ and enter them on the Results Page. Also enter the resultant sum. If you were careful in carrying out the experiment you should find that the predicted value differs from the measured value by no more than a few percent.

While admiring your results, keep in mind that the stretching out of the collision time has nothing to do with the friction in the bearing. It is due to the time necessary for the washer and rotor to reach a common angular velocity. This process is carried out by forces internal to the system under consideration, thus it does not change the angular momentum of the system. The external torque, that due to the friction in the bearing, simply has a longer time to act on the system than it does in the fast collision; thus it will produce a larger percentage correction to the result one would expect if the angular momentum of the system were conserved.
A Fast Inelastic Collision:

Keep the same settings for the LabView program and this time drop a washer hook side down. You should see a graph like this one.

When you have a collision you like, save it as AMfastCollide then open the file in Excel. Proceed to analyze the data exactly the same way you did for the slow collision. Here, though, not only are $t_1$ and $t_2$ much closer together, but there seem to be strange things going on at the beginning and the end of the collision. These effects are due to a slight “springyness” in the interaction between the hooks and fur of the velcro. We will not pursue these effects here. Simply do your best to find the times and angular velocities where the extrapolation of the slow changes in $\omega$ meets the extrapolation of a line through the rapid fall of $\omega$. As before, enter the results of the analysis as you go along on the Results Page.
Results Page for Experiment 4

**MOMENT OF INERTIA**

- $\alpha_1$
- $\alpha_2$ Average $\omega$ while finding $\alpha_2$
- $I_R$
- $\tau_f$ Model prediction for $\tau_f$

**VARIATION OF FRICTION TORQUE**

Model expression for $\tau_f(\omega)$

**FAST COLLISION**

- $t_1$ $\omega_1$ $\frac{\omega_1 + \omega_2}{2\tau_f}$
- $t_2$ $\omega_2$ $\tau_f$

$\omega_2$, no $\tau_f$ + $\tau_f$ correction = $\omega_2$ predicted

**SLOW COLLISION**

- $t_1$ $\omega_1$ $\frac{\omega_1 + \omega_2}{2\tau_f}$
- $t_2$ $\omega_2$ $\tau_f$

$\omega_2$, no $\tau_f$ + $\tau_f$ correction = $\omega_2$ predicted
Experiment 4: Material to be Handed In

CLASS

TABLE GROUP

NAME

NAME

NAME

NAME

Please fill out the attached Results Page representing your joint efforts in class.

Each person should also attach the required materials from PreLab done out of class.