Purpose of the Experiment
The goal of this experiment is to study the spring force and centripetal acceleration. You will observe a round weight attached to a rotating shaft by a spring; you will adjust the angular velocity $\omega$ of the shaft rotation and measure the radius $r_m$ of the circular motion.

You will do the following things in this experiment:

- Measure the radius $r_m$ of the orbit of the mass $m$ as a function of $\omega$ for the shaft, and interpret your measurements with a model based on two things (i) the magnitude of the centripetal force is $m v^2 r_m$ ($\omega = \frac{v}{r_m}$ is the angular speed in radians per second), and (ii) the spring obeys Hookes’ law (spring force proportional to amount of stretch). To simplify the analysis, assume that the mass of the spring can be neglected.

- This model predicts $r_m \to \infty$ at a critical angular frequency, $\omega_c$, for the shaft rotation; you will observe the approach to this behavior and understand what causes it.

A checklist of steps to follow is included at the end of this document. Each group will be given a printed copy of this list during class.

Description of the Model
First, assume that the spring obeys Hookes’ law, $|F_{\text{spring}}| = k(r_m - r_0)$, where $r_0$ is the position of the mass when the spring is unstretched. For the mass to go in a circle of radius $r_m$, there must be a so-called centripetal force, $|F_{\text{cent}}| = m \omega^2 r_m = m r_m \omega^2$, where $\omega$ is defined above. Assuming that the force of the spring is what keeps the ball moving in a circle, we can set the two equations for the force equal to each other and solve for the radius of the motion, $r_m$, as a function of the angular speed, $\omega$. Solving this yields:

$$r_m(\omega) = \frac{r_0}{1 - m \omega^2 / k} = \frac{r_0}{1 - (\omega/\omega_c)^2} \text{ where } \omega_c = \sqrt{k/m}$$

(1)

You can use the computer program to plot $r_m$ versus $\omega$ (the program calls it “R vs omega”), and fit your measurements using Equation 1.

Experimentally Determining the Spring Force Law
In deriving Eq. (1), we assumed both $|F_{\text{cent}}| = m r_m \omega^2$ and $|F_{\text{spring}}| = k(r_m - r_0)$. But, suppose you don’t know anything about your spring. In this case, you can assume only that $|F_{\text{cent}}| = m r_m \omega^2$, and use this to calculate the force exerted by the spring using
\[ F_{\text{spring}} = F_{\text{cent}} \]. Thus, you could experimentally find \( F_{\text{spring}} \) as a function of \( r_m \) to examine the spring force law explicitly. You will study this using a fit to a plot of \( F_{\text{spring}} \) vs \( r_m \).

**An Interesting Instability**

Eq. (1) makes an interesting prediction: that at a critical angular velocity \( \omega_c = \sqrt{k/m} \) the system becomes unstable (the denominator of Eq. (1) goes to zero so the predicted value of \( r_m \) becomes infinite) and the mass will fly off to infinity. It is fairly easy to see why this happens. Both the centripetal force required to keep the mass in a circular orbit, \( |F_{\text{cent}}| = m r_m \omega^2 \), and the restoring force \( |F_{\text{spring}}| = k (r_m - r_0) \) from the stretched spring increase with radius \( r_m \). Normally, the spring stretches just enough to provide the required centripetal force and the mass settles into a stable circular motion. However, if \( \omega \geq \omega_c \), \( F_{\text{cent}} \) increases more rapidly with \( r_m \) than \( F_{\text{spring}} \) does. If the spring stretches to provide more restoring force, it only makes the situation worse! Not surprisingly, we will not explore this possibility experimentally. However, you will be able to calculate how close you came to this critical rotational speed.

**Procedure**

The program to run is called Circular-Motion. Open the Desktop folder labeled “Student’s Home” and then the folder labeled “8.01 Labs”. As usual, the program is controlled by an action menu at the upper left.

The first thing you are asked to do is enter the radius when the ball is not spinning. Use a value of 0.045 (4.5 cm), not the 0.046 shown in the picture.

When you have the ball spinning, the program will automatically count and time the rotations of the ball and calculate the \( \omega \). The picture above shows the peaks that the program records and uses to make this calculation. How it works is described below. All you need to do is verify that the program counted the periods correctly and enter the radius at which the ball is rotating.

You find the radius by looking down through a movable tube with a pointer that you align with the center of the ball. The other end of this pointer has a scale which you use to find the radius of the circular motion. To help with the alignment, a white light comes on while the ball rotates.
Examples of data analysis

Hookes’ Law (force depends linearly on amount of stretch) applies exactly only to an ideal spring. Real springs may or may not have a linear relationship or may be close to linear for some range of forces but deviate significantly for larger forces. Every spring must deviate at some point, if for no other reason than the spring breaks if you pull hard enough. The first step in the analysis is to check whether or not your spring has a linear force-stretch dependence. You can use the Plot Control to select either “F vs R” to look at the spring force directly or “R vs Omega” to investigate Eq. (1). The program also allows you to fit both functions.

When you choose “F vs. R” from the Plot control pull-down menu, the program calculates the centripetal force from

$$F_{\text{cent}} = m \frac{v^2}{r_m} = m r_m \omega^2$$

and then plots this force versus the length of the spring. Since the force to move the ball in a circle comes only from the spring, you can examine this plot to learn the characteristics of the force exerted by the spring as it is stretched. The data points can be fit to a straight line ($|F_{\text{spring}}| = k(r_m - r_0)$) to find the spring constant, $k$, and unstretched length, $r_0$. The graph at the left is an example of a fit.

Typically, you will see that the point at $F = 0$ does not seem to lie on the best fit straight line. In fact, it often looks as if $F_{\text{spring}} > 0$ when $r = r_0$. That is in fact the case. In one spring manufacturer’s catalogue (http://www.centuryspring.com) you will find that small springs like the ones in our experiment are typically wound with an initial tension of a few tenths of a Newton. In practice, this effect is very small and not clearly visible in all cases.

If you plot “R vs omega”, the program will plot the radii you entered versus the measured $\omega$ and allow you to fit Eq. (1) to your measurements. This allows you to test the prediction that the radius begins to increase very rapidly at $\omega$ approaches a certain critical value. You will decide if the assumptions of the model are confirmed by your measurements by looking at this best fit function plotted on the graph. One example of a fit of R vs Omega is shown at left. Notice that for larger values of $\omega$, the radius begins to increase very rapidly.

More Details:

You will use the LabVIEW program CircularMotion to measure the rotation period, $T$, of the mass, and from that calculate the angular rotation frequency $\omega = 2\pi/T$. As the motor rotates, a magnet in the counterweight triggers a reed relay on the top cover of the box and makes a voltage pulse every time the magnet passes under it. The voltage is normally close to 0, but rises to between 3 and 4 V when the magnet closes the reed relay. The program CircularMotion acts like an oscilloscope to detect these voltage pulses, counts the number in a known period of time and calculates the angular frequency of rotation, $\omega$, of the motor shaft. The “Run Scope” option will start the program measuring the voltage generating a plot like the one in the graph above. The “Record r, omega” option will analyze the scope trace to determine $\omega$ and open a window asking you to type in the radius $r_m$ that you measured.
**Experiment 2: Uniform Circular Motion**

**Setting up the apparatus:**
If not already done, you need to attach the spring and ball to the hook on the central shaft. One side of the apparatus slides open. **Don’t forget to close the side panel before starting the ball spinning.** The weight moves at fairly high speed and could give a nasty whack to your finger if you are careless.

**Electronic setup:**
1. The 750 interface should be powered on (green light comes on).
2. Make sure the knob on the equipment is turned down (counter-clockwise until it clicks) before connecting the experimental apparatus to the power.
3. The apparatus should be plugged into power and plugged into the 750 input A. Turn the knob clockwise to start the rotation and try different speeds to see how it behaves.
4. The CircularMotion program can be found by opening the Desktop folder labeled “Student’s Home” and then the folder labeled “8.01 Labs”.
5. If you get an error, something is turned off or unplugged, so you need to check your setup first.

**To take data:**
1. Turn the rotation **off**.
2. Under Action Menu (see picture in writeup), select “Record r,omega”. A window will pop up in which you should enter 0.045 (4.5 cm) in the box for the radius (see picture in writeup). Ignore the other boxes. This is $r_0$, the radius when the spring is unstretched. Later, the program will fit this to get a more accurate value. You only need to do this once.
3. Hit “OK”.
4. Start the ball spinning by turning the knob clockwise. The white LED should light, illuminating the ball once you align the viewing tube.
5. Let the system stabilize for about 30 seconds. It will help if one student holds the box steady so it doesn’t vibrate.
6. Look down through the tube and measure the radius by aligning the nail inside the tube with the black bar through the middle of the ball. The nail on the outside next to the scale gives you the reading.
7. Under Action Menu, select “Run Scope”. Hit the “‘RUN” button. A series of peaks will appear on the plot (see picture in writeup).
8. Under Action Menu, select “Record r,omega” and enter your value for the radius in the box. Be careful to **get the units correct**. The program uses the scope data to find the $\omega$ automatically (see picture in writeup). **However**, noise in the data frequently causes the program to count the **wrong** number of periods. The number of periods should be the number of peaks shown minus 1. Notice that the picture in the write-up has 7 clean peaks and the readout box shows 6 periods. If the program miscounted, enter the correct number of periods in the box.
9. **Warning:** Re-read the previous item about checking the number of periods!
10. Repeat for at least 5-6 different speeds. You want values of $r$ between 5 and 10 cm. You don’t need to do them in any particular order, you can go up and down.
To analyze your data:

1. Go to the Plot Control and select either “F vs R” or “R vs Omega” depending on which questions you are answering and hit “Replot Now”.
2. If you see any strange points, you may want to delete them or correct them. The most common mistake is entering a radius in the wrong units. See the section below about Editing for instructions.
3. On the Results tab, select the option to fit \( r_0 \) as a parameter. You can experiment with a fixed \( r_0 \) to see the difference but we will not use those fits.
5. The boxes on the Results tab will show the fitted values as well as the \( \chi^2 \) which is a measure of the quality of the fit. A value close to or less than 1.0 indicates a reasonably good fit.

Instructions for expanding part of the plot (for Bonus Question 8)

The writeup for the Projectile Motion Experiment gave some instructions for expanding part of a plot. The basic procedure is to use the cursor control at the upper left of the plot. Normally, this is on the cross-hair (at far left as shown) which allows you to move the cursors.

To expand part of the plot, select expand (center icon with magnifying glass). Press left mouse button, then select horizontal or vertical expand (top center and top right icon in submenu), then release mouse button. Put cursor (now like a magnifying glass) to one side of the desired region, press left mouse button, hold and move to the other side of the region, release mouse button. Region you selected expands to fill the plot. To go back, under expand (magnifying glass icon), select full screen (bottom left icon in submenu). You should see the full plot on your screen again.

Editing to remove mistakes in recording data

To edit the data which are in the table, click on any of the cells and edit the entry. You can also right-click with the cursor over a table entry and get a menu (see the one to the right) that will let you do other things, such as delete rows (or even delete everything in the table so be careful).

If you want the edited table contents to replace the data that are stored in the computer’s memory, choose “Update from Table” from the Action menu (Warning: This cannot be undone). Then you should update the graph by clicking “Replot Now”. Now you can fit the new data.

If you change your mind, you can restore the table as it was before editing by clicking “Replot Now”, as long as you have not selected “Update from Table”.

Note that this only updates the table you are using (either “F vs R” or “R vs omega”). If you want to delete the same bad point from the other table you need to go to that plot and repeat these steps.