Experiment 6: Angular Momentum

Purpose of the Experiment:
In the first part of this experiment, you investigate rotational collisions and the conservation of angular momentum. This is the rotary counterpart of the earlier experiment in which you investigated linear collisions. In the second part of the experiment, you will test your understanding of the relationship between torque, moment of inertia, and angular acceleration.

Theory:
The motor and axle are very light and close to the rotation axis so the moment of inertia of the whole rotating system is, to a very good approximation, simply a constant times the number of washers. When the dropped washer or washers are not rotating as they are dropped, angular momentum conservation predicts:

\[ I_{WasherSpin} \times \omega_1 + I_{WasherDrop} \times 0 = (I_{WasherDrop} + I_{WasherSpin}) \times \omega_2 \]

so that:

\[ \omega_2 = \omega_1 \frac{I_{WasherSpin}}{I_{WasherDrop} + I_{WasherSpin}} = \omega_1 \frac{N_{WasherSpin}}{N_{WasherDrop} + N_{WasherSpin}} \]

where \( N_{WasherSpin} \) and \( N_{WasherDrop} \) are the numbers of washers initially spinning and dropped, respectively.

Now, consider the motor simply slowing down without a collision. If the slowing torque, \( \tau_{slow} \), is constant, then the angular acceleration, \( \alpha = \frac{\tau_{slow}}{I_{Tot}} \), should be inversely proportional to the moment of inertia (i.e. \( \alpha \) should vary inversely with the number of washers). If \( \tau_{slow} \) is not a constant, the angular acceleration will behave differently. One possibility is that the torque is “friction-like”, i.e. it depends on the normal force which, in turn, depends on the weight of the washers so \( \tau_{slow} \propto N_{Washer} \). In this case, both the torque and moment of inertia would rise linearly with the number of washers and the angular acceleration would be close to a constant as more washers are added.

The heart of the experiment is a high quality DC motor which spins a rotor up to several hundred radians per second. When you hold down the red pushbutton switch on the apparatus, power is applied to the motor; when you release it, the rotor coasts and the motor output voltage can be read by the computer. When power to the motor is shut off, it serves as a tachometer-generator whose output voltage is proportional to the angular velocity of the rotor; thus the angular velocity of the rotor can be determined by measuring this output voltage.

The apparatus looks like this:

You will use the AngularMomentum program. If you get an error, something is turned off or unplugged, so you need to check your setup first.
Collision:
One of the washers has a smooth (brass) side and the “hooky” half of some Velcro glued to the other. If you drop it Velcro side down onto the rotor, it will stick almost instantly, resulting in a collision that lasts only a few ms.

The angular momentum is constant if there are no external torques acting on a system. Unfortunately, the system consisting of the rotor and the dropped washer has an external stopping torque acting on it. However, if the collision is very fast, the angular impulse of the stopping torque will be small enough that it may be ignored; in that case, angular momentum should be constant during the collision. You will test that by studying three fast collisions.

Taking Data:
The default values of the following parameters (visible using the Data Collection option under the Experiment pull-down menu) should work well: data collection length = 4 s and sampling rate = 200 Hz (see image right). Hold the washer(s) level and centered just above the rotor with the Velcro side down. Spin the rotor up by holding the red button down, release the button, click START, let the rotor coast for between one and two seconds, and drop the washer. You will get somewhat better results if you do not spin the rotor up to its maximum speed (less vibration).

At the end of the 4 s measuring period, you should get a graph like the one to the right. You can use the zoom control to expand the graph to show more detail for the collision.

The LabPro device measures voltage, but in this experiment we want to analyze angular velocity. A conversion factor of 70 rad/s/V was found experimentally and should be used throughout this lab to obtain angular velocity.
Finding Numbers:
You need the angular speed ($\omega$) before and after the collision. Examine the graph of $\omega$ vs. time and use the cursor to read off the value of $\omega$ (the Y coordinate) just before and after the collision. Fit a simple line to the data taken before the collision by highlighting the data with your mouse and selecting Analyze>Linear Fit. The form of the fit will be $\omega = A + Bt$. The result of the fit will be drawn over the data; expand the graph with the zoom controls to see the fit more clearly. You can see the numerical results of the fit displayed in a box within the plot. After recording these results, fit a line to the data after the collision and record the fit parameters.

Theory of the Extra Parts
When one dropped washer isn’t moving, angular momentum conservation predicts $I_{\text{washer}} \times \omega_1 + I_{\text{washer}} \times 0 = 2 \times I_{\text{washer}} \times \omega_2$ and $\omega$ should drop by a factor of $\approx 2$. If the dropped washer is spinning in the same direction, the second term on the left of the equation will be positive and so $\omega_2$ will be larger than $\omega_1/2$. In contrast, if the dropped washer is spinning in the opposite direction, the second term on the left of the equation will be negative and so $\omega_2$ will be smaller than $\omega_1/2$.

For the washer dropped smooth side down, the friction between the two washers is much less so it takes a lot more time for the dropped washer to accelerate up to the same speed as the bottom washer. The ratios of $\omega$ and $\alpha$ should be about a factor of 2 again and the only difference is the duration time of the collision. You should get a graph like the one to the right.

The graph, of course, shows only the angular velocity of the rotor, but you can see the collision lasts much longer than the fast collision (about 440 ms), there is some indication of bouncing and wobbling of the washer, and the washer slides on the rotor until the two bodies have matched rotation speeds. The collision is over as soon as the two rotation speeds are the same. During the collision $\omega$ decreases close to linearly (constant $\alpha$, which means a constant torque on the rotor). For fun, you can find $\alpha_{\text{rotor}}$ during the collision by fitting the data between the cursors.
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Some of these steps have been done for you but check the setup.

Electronic setup:

1. The Lab Pro interface should be powered on (green light comes on).
2. Plug the rotary motion apparatus into its power supply; you should see the LED in the plastic pipe elbow come on.
3. Connect the generator output voltage (the jacks farthest from the power input connector) to channel 1 on the Lab Pro interface. This version of the experiment will have only the power and two other plugs, not 4 as shown in the picture.
4. Open the AngularMomentum program by clicking the Logger Launcher icon on the Desktop and navigating to 801L_Templates>AngularMomentum. If you get an error message, something is turned off or unplugged, so you need to check your setup.

Taking Data:

1. One of the washers has a smooth (brass) side and the “hooky” half of some Velcro glued to the other. If you drop it Velcro side down onto the rotor, it will stick almost instantly, resulting in a collision that lasts only a few ms.
2. Set the sample rate to 200 Hz but the run time to 4 s (see picture in writeup).
3. Hold the washer(s) level and centered just above the rotor with the Velcro side down.
4. Spin the rotor up by holding the red button down, release the button, click “Collect”, let the rotor coast for between one and two seconds, and drop the washer.
5. You will get somewhat better results if you do not spin the rotor up to its maximum speed (less vibration).
6. You need the angular speed ($\omega$) before and after the collision. The sensor measures voltage, which is converted into ($\omega$) by a conversion factor of 70 rad/s/volt. This is automatically plotted for you.
7. Select Analyze>Examine. Use the cursor to read off the value of $\omega$ (the vertical coordinate) just before and after the collision.
8. Repeat by dropping one washer onto two spinning washers and two washers onto one spinning washer.
9. For the second half of the experiment, put on one, two, or three washers, spin up the motor, let it coast for a few seconds, then take a measurement. Your data will simply be a single (approximately) straight line.
10. You can obtain the angular acceleration, $\alpha_1$, from the slope of the graph. Fit a straight line to the data before the collision by highlighting the data range and selecting Analyze>Linear Fit.
11. The result of the fit will be drawn over the data; expand the graph with the zoom controls to see the fit more clearly.
12. You can see the numerical results of the fit displayed in a box within the plot.
13. Perform a linear fit to the data after the collision, as well.