Experiment 3: Momentum and Collisions

Purpose of the Experiment:
In this experiment you collide a cart with a spring that is attached to a force sensor and also collide one moving cart with a second initially stationary cart on a level track. You will measure the velocity of one cart and the force exerted by the spring while it is compressed. You will analyze your data to investigate the following things from this experiment:

- The correspondence between impulse and change in momentum.
- The predictions derived using conservation of momentum in inelastic collisions as well as momentum and energy for elastic collisions.

When a net external force acts on a moving object, it will change the momentum of the object. The basic equation can be written in two equivalent ways: \( \frac{dp}{dt} = F \) or \( \Delta \vec{p} = \int F dt \), where \( \Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} \). You will test the integral formula by measuring both the velocity before and after a spring acts on a cart and the integral of the force of the spring as a function of time. The computer program will integrate the force data numerically.

The force sensor with a spring is mounted at one end of the track and the motion sensor is clipped to the other end. To balance all external forces along the track, it should be level, but since you will measure speed just before and just after collisions, it is not critical that the track be perfectly level.

When no net external force acts on an object or group of objects, the total momentum is a constant. So-called “internal” forces, such as those between objects in a collisions, do not change the total momentum. The collisions on the track are one dimensional so we only need to worry about one component of the momentum vector. If the track is level and friction can be neglected, there are no net external forces. If a moving cart hits a stationary cart and the two stick together after the collision, the relevant momentum equation is \( m_1 v_0 = (m_1 + m_2) v_1 \). If a moving cart hits a stationary cart and the two do not stick together after the collision, the relevant momentum equation is \( m_1 v_0 = m_1 v_1 + m_2 v_2 \) which has two unknowns, so you need more information to solve the problem. If the collision is also known to be elastic (i.e. KE is conserved), the additional equation is \( \frac{1}{2} m_1 v_0^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \). These two equations can be combined and solved to find \( v_1 = \frac{m_1 - m_2}{m_1 + m_2} v_0 \) and \( v_2 = \frac{2m_1}{m_1 + m_2} v_0 \). In this experiment, you will measure the velocity of the incoming cart before \( v_0 \) and after \( v_1 \) inelastic and elastic collisions and attempt to verify two of these three equations for final velocities. If you have time left, the optional part also checks the equation for the speed of the struck cart \( v_2 \) in an elastic collision.
Data is taken with the *Momentum* program. You will use the “Collect” and “Integral” functions.

**Momentum and Impulse:** In this part, you push one cart so it bounces off of the spring. **Be sure to tare (zero) the force sensor before each measurement. Do not tare the motion sensor.** This can be done by clicking the “Zero” button or going to `Experiment>Zero`. The program records position, velocity, acceleration, and force automatically, but will only plot velocity and force. You may insert more plots if you wish. For velocity vs. time, you should see something like the figure at the right. You can move the cursor to find the speed just before and just after the cart hits the spring as shown. It might help to expand the scale.

To measure the integral of Force versus time (i.e. the impulse), use the plotting controls to look at the force plot. Click and highlight a region of data and select the integral symbol at the top of the window (or choose `Analyze>Integral`). Move the brackets on the graph to change the selected data range. An expanded version of the force plot after the fit is shown in the left figure below with the area integrated shaded in red.
Collisions

Don’t let the carts hit the motion sensor!
For inelastic collisions, arrange the carts with the Velcro pads facing each other. You may
decide to put weights on one or both of the carts. The target cart should be placed on the
track with the end facing the incoming cart about 50-60 cm from the motion sensor. If you
push too hard a cart may jump off the track during the collision. You should experiment to
get this right before you start to make measurements. For inelastic collisions, your velocity
plot should look something like the plot at the right. Use the cursor to read off the velocities
just before and just after the collision.

For elastic collisions, arrange the carts with the blank sides facing each other. The carts
have opposing magnets so they repel each other. You need to decide what weights to put
on one or both of the carts. In the first part, the incoming cart should be lighter and in
the second part, it should be heavier. As before, the target cart should be placed on the
track with the end facing the incoming cart about 50-60 cm from the motion sensor. For
the lighter incoming cart, your velocity plot will look similar to the bouncing off the spring
experiment (see figure at the top of page 2), while for the heavier incoming cart it will
resemble the inelastic velocity plot (see figure at the bottom of page 2). Use the cursor to
read off the velocities of the incoming cart just before and after the collisions.

Elastic Collision: Optional Additional Analysis
Under the assumption that the cart hits the spring and rebounds with the same speed
(which you can evaluate and perhaps correct for using your data from the first part of this
experiment), the change in momentum of the struck cart can be used to calculate the speed
from: $\int F \, dt = 2m_2 v_2$, where $\int F \, dt$ is the magnitude of the impulse (i.e. the integral of Force
versus time). Measuring this impulse allows you to calculate the velocity of the struck cart
after the collision. The speed of $m_2$ after the collision can be used to study these questions:

While the carts were colliding, did the momentum change?

While the carts were colliding, did the total kinetic energy change?
Appendix: Magnetic Force

This is not part of the experiment you are asked to do, but the result is interesting. When the carts collide elastically, they exert forces on each other by the repulsion between two pairs of magnets. The force function between the two carts was found to look like the plot at the right. The solid line is a fit by the function $|F| = Ae^{-x/\ell}$, where $x$ is the separation between the carts and $\ell$ is a characteristic length associated with the magnets. Note that this force is very different from an ideal compressed spring, $|F_{\text{spring}}| = k|\ell - \ell_0|$.

There is some systematic deviation visible, but it is a pretty good approximation to the force law between the carts as they collide.

If we define the positive $\hat{x}$ direction to point in the direction of the magnet force, then it is given by $\vec{F}_{\text{mag}} = Ae^{-x/\ell}\hat{x}$, and an external force of $\vec{F}_{\text{Ext}}(x) = -Ae^{-x/\ell}\hat{x}$ is required to push the carts together. If the carts start very far apart (an infinite distance), then the work done by that external force to bring them to a separation distance $d$ is:

$$W = \int_\infty^d F_{\text{Ext}}(x) \cdot dx = -A \int_\infty^d e^{-x/\ell} \, dx = A\ell e^{-x/\ell} \bigg|_\infty^d = A\ell e^{-d/\ell}$$

You can check this result by taking the derivative of the next to last term. Note that since the carts repel each other, you need to do positive work to push them together. This might make a good exam question :)
Experiment 3: Momentum and Collisions

Electronic setup:
1. The Lab Pro interface should be powered on (green light comes on).
2. Connect the motion sensor and the force sensor to the DIG/SONIC 1 and channel 1 of the Lab Pro interface, respectively.
3. Be sure to tare (zero) the force sensor before each measurement (button on the side).
4. The program to run is called Momentum. Open the Desktop folder labeled “Student’s Home” and then the folder labeled “8.01L Labs.”
5. If you get an error, something is turned off or unplugged, so you need to check your setup first and ask for help if it still doesn’t work.

Mechanical setup:
1. The motion sensor should be aimed slightly below the center of the cart rather than pointing directly at it.
2. The slide switch on top of the motion sensor should be set to the narrow beam position.
3. The force sensor should be placed at the end of the track against the back-stop. It should be set to the ±10 N range.
4. Your track should be level. If necessary, you can improve it using the level adjustment screw. The best way to test this by looking at the velocity data for a cart running from one end of the track to another. Ideally, the velocity will be a constant. Since we will be measuring speed just before and just after collisions, it is not that critical that the track be perfectly level.

To take data:
1. Make sure Data Collection parameters are set to the following default values: length=4s and sampling=1000 Hz. These settings are found under Experiment>Data Collection.
2. In this Logger Pro program you will use the “Collect” and “Integral” functions.
3. When you are ready to collect, hold the cart or carts in position, choose “Collect,” and launch the incoming cart.
4. If you wave your hands around the carts as they move, you may confuse the motion sensor. Try to keep your hands away during the measurement.

Momentum and Impulse:
1. Don’t let the rebounding cart hit the motion sensor!
2. In this part, you push one cart so it bounces off of the spring. Do not put weights on the cart.
3. Be sure to tare (zero) the force sensor before each measurement.
4. To find the speeds, examine the velocity plot. You should see something like the picture in the writeup (top of page 2).
5. Click the “Examine” button or go to Analyze>Examine in order to get a cursor on your graph.
6 Move the cursor to find the speed just before and just after the cart hits the spring. It might help to expand the scale.
7 If you have forgotten how to expand the scale and move the cursors, see the link to “Logger Pro Hints” on the 8.01L Experiment page.
8 Now, you need to measure the integral of the Force versus time plot. Select a data range within the force plot and click the “Integral” button at the top of the window (this can also be found in Analyze>Integral).
9 An expanded version of the force plot after the fit is shown in the writeup with the integrated area shaded in red.

Inelastic Collision

1 Don’t let the rebounding carts hit the motion sensor!
2 Arrange the carts with the Velcro pads facing each other. You may decide to put weights on one or both of the carts.
3 The target cart should be placed on the track with the end with the Velcro pads about 50-60 cm from the motion sensor.
4 If you push too hard a cart may jump during the collision. You should experiment to get this right before you start to make measurements.
5 An example of what your velocity plot should look like is in the writeup (bottom of page 2).
6 Use the cursor to read off the velocities just before and just after the collisions.

Elastic Collision

1 Don’t let the rebounding carts hit the motion sensor!
2 Arrange the carts with the blank sides facing each other. The carts have opposing magnets so they repel each other.
3 You need to decide what weights to put on one or both of the carts. In the first part, the incoming cart should be lighter and in the second part, it should be heavier.
4 The target cart should be placed on the track with the end facing the incoming cart about 50-60 cm from the motion sensor.
5 If you push too hard a cart may jump during the collision. You should experiment to get this right before you start to make measurements.
6 For the lighter incoming cart, your velocity plot will look similar to the bouncing off the spring experiment, while for the heavier incoming cart it will resemble the inelastic collision velocity plot.
7 Use the cursor to read off the velocities just before and after the collisions.

Optional: Additional Elastic Collision Analysis:
If you finish early, you can do additional analysis of an elastic collision by using the spring impulse to find the speed of the second cart after the collision. See writeup for details.