E04-1

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 4: RC Circuits

OBJECTIVES

1. To explore the time dependent behavior of RC Circuits
2. To understand how to measure the time constant of such circuits

PRE-LAB READING

INTRODUCTION

In this lab we will continue our investigation of DC circuits, now including, along with our “battery” and resistors, capacitors (RC circuits). We will measure the relationship between current and voltage in a capacitor, and study the time dependent behavior of RC circuits.

The Details: Capacitors

Capacitors store charge, and develop a voltage drop $V$ across them proportional to the amount of charge $Q$ that they have stored: $V = Q/C$. The constant of proportionality $C$ is the capacitance (in Farads = Coulombs/Volt), and determines how easily the capacitor can store charge. Typical circuit capacitors range from picofarads (1 pF = $10^{-12}$ F) to millifarads (1 mF = $10^{-3}$ F). In this lab we will use microfarad capacitors (1 μF = $10^{-6}$ F).

RC Circuits

Consider the circuit shown in Figure 1. The capacitor (initially uncharged) is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch $S$ is closed.

![Figure 1](a) RC circuit (b) Circuit diagram for $t > 0$

In class we derived expressions for the time-dependent charge on, voltage across, and current through the capacitor, but even without solving differential equations a little
thought should allow us to get a good idea of what happens. Initially the capacitor is uncharged and hence has no voltage drop across it (it acts like a wire or “short circuit”). This means that the full voltage rise of the battery is dropped across the resistor, and hence current must be flowing in the circuit \((V_R = IR)\). As time goes on, this current will “charge up” the capacitor – the charge on it and the voltage drop across it will increase, and hence the voltage drop across the resistor and the current in the circuit will decrease. This idea is captured in the graphs of Fig. 2.

\[
\begin{align*}
V_f &= \epsilon \quad Q_f = C \epsilon \\
V_{Capacitor}, V_{Capacitor} & \text{Time} \\
VR, 0 &= \epsilon \\
I_0 &= \epsilon / R \\
V_{Resistor}, I & \text{Time}
\end{align*}
\]

Figure 2  (a) Voltage across and charge on the capacitor increase as a function of time while (b) the voltage across the resistor and hence current in the circuit decrease.

After the capacitor is “fully charged,” with its voltage essentially equal to the voltage of the battery, the capacitor acts like a break in the wire or “open circuit,” and the current is essentially zero. Now we “shut off” the battery (replace it with a wire). The capacitor will then release its charge, driving current through the circuit. In this case, the voltage across the capacitor and across the resistor are equal, and hence charge, voltage and current all do the same thing, decreasing with time. As you saw in class, this decay is exponential, characterized by a time constant \(\tau\), as pictured in fig. 3.

\[
\begin{align*}
V_{R,0} &= V_{C,0} = \epsilon; \\
I_0 &= \epsilon / R; \\
Q_0 &= C \epsilon
\end{align*}
\]

\[
\begin{align*}
VR, V_C, I, Q & \text{Time} \\
VR, 0 = V_0; \\
0.368 V_0 & \quad \text{Time}
\end{align*}
\]

Figure 3 Once (a) the battery is “turned off,” the voltages across the capacitor and resistor, and hence the charge on the capacitor and current in the circuit all (b) decay exponentially. The time constant \(\tau\) is how long it takes for a value to drop by \(e\) (~2.7).
The Details: Measuring the Time Constant $\tau$

In this lab you will be faced with an exponentially decaying current $I = I_0 \exp(-t/\tau)$ from which you will want to extract the time constant $\tau$. We will do this in two different ways, using the “two-point method” or the “logarithmic method,” depicted in Fig. 7.

In the two-point method (Fig. 7a) we choose two points on the curve $(t_1, I_1)$ and $(t_2, I_2)$. Because the current obeys an exponential decay, $I = I_0 \exp(-t/\tau)$, we can extract the time constant $\tau$ most easily by picking $I_2$ such that $I_2 = I_1/e$. We should, in theory, be able to find this for any $t_1$, as long as we don’t switch the battery off (or on) before enough time has passed. In practice the current will eventually get low enough that we won’t be able to accurately measure it. Having made this selection, $\tau = t_2 - t_1$.

In the logarithmic method (Fig. 7b) we fit a line to the natural log of the current plotted vs time and obtain the slope $m$, which will give us the time constant as follows:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\ln(I(t_2)) - \ln(I(t_1))}{t_2 - t_1} = \frac{1}{t_2 - t_1} \ln\left(\frac{I(t_2)}{I(t_1)}\right)$$

$$= \frac{1}{t_2 - t_1} \ln\left(\frac{I_0 e^{-t_2/\tau}}{I_0 e^{-t_1/\tau}}\right) = \frac{1}{t_2 - t_1} \ln\left(e^{-(t_2-t_1)/\tau}\right) = \frac{1}{t_2 - t_1} \left(-\frac{(t_2-t_1)}{\tau}\right) = -\frac{1}{\tau}$$

That is, from the slope (which the software can calculate for you) you can obtain the time constant: $\tau = -1/m$.

In using both of these methods you must take care to use points well into the decay (i.e. not on the flat part before the decay begins) and try to avoid times where the current has fallen close to zero, which are typically dominated by noise.
APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the 750 interface to create a “variable battery” which we can turn on and off, whose voltage we can change and whose current we can measure.

2. AC/DC Electronics Lab Circuit Board

We will also again use the circuit board of Fig. 8. This time we will use the inductor (E) as well as the connector pads (F) for resistors and capacitors, and the banana plug receptacles in the right-most pads to connect to the output of the 750.

![Image of AC/DC Electronics Lab Circuit Board](image)

**Figure 8** The AC/DC Electronics Lab Circuit Board, with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads

3. Current & Voltage Sensors

Recall that both current and voltage sensors follow the convention that red is “positive” and black “negative.” That is, the current sensor records currents flowing in the red lead and out the black as positive. The voltage sensor measures the potential at the red lead minus that at the black lead.

![Image of Current Sensors](image) ![Image of Voltage Sensors](image)

**Figure 9** (a) Current and (b) Voltage Sensors
4. Resistors & Capacitors

We will work with resistors and capacitors in this lab. While resistors (Fig. 10a) have color bands that indicate their value, capacitors (Fig. 10b) are typically stamped with a numerical value.

Figure 10 Examples of a (a) resistor and (b) capacitor. Aside from their size, most resistors look the same, with 4 or 5 colored bands indicating the resistance. Capacitors on the other hand come in a wide variety of packages and are typically stamped both with their capacitance and with a maximum working voltage.

GENERALIZED PROCEDURE

This lab consists of two main parts. In each you will set up a circuit and measure voltage and current while the battery periodically turns on and off.

Part 1: Measuring Voltage and Current in an RC Circuit
In this part you will create a series RC (resistor/capacitor) circuit with the battery turning on and off so that the capacitor charges then discharges. You will measure the time constant using both methods described above and use this measurement to determine the capacitance of the capacitor.

Part 2: Measuring Voltage and Current in an RC Circuit
In this part you will add a second resistor in parallel with the capacitor to confirm your understanding of the in class problem worked before this part of the lab.

END OF PRE-LAB READING
IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose “Save Target As” to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.

2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface. We will obtain the current directly from the “battery” reading.

3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

MEASUREMENTS

Part 1: Measuring Voltage and Current in an \( RC \) Circuit

1. Quickly measure the resistance of the resistors (how can you do that?)
2. Create a circuit with the first resistor and the capacitor in series with the battery
3. Connect the voltage sensor (channel A) across the capacitor
4. Record the voltage across the capacitor \( V \) and the current sourced by the battery \( I \) (Press the green “Go” button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 1:

What is the resistance of the resistor? Using the two-point method, what is the time constant of this circuit? Using this time constant and the typical expression for an RC time constant, what is the capacitance of the capacitor?

Question 2:

Using the logarithmic method, what is the time constant of this circuit? Using this time constant, what is the capacitance of the capacitor?
Part 2: Measuring Voltage and Current in a parallel RC Circuit

1. Add the second resistor in parallel with the capacitor
2. Record the voltage across the capacitor $V$ and the current sourced by the battery $I$ (Press the green “Go” button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 3:
Using one of the two methods used above, what is the time constant of this new circuit? Is there any difference between this circuit (where the battery "turns off") and the one you solved analytically in class (where a switch opens next to the battery)? If so, what? If not, why not?

Further Questions (for experiment, thought, future exam questions…)

- What happens if we instead put the second resistor in series with the capacitor?
- What if we change the order of the elements in the circuit (e.g. put the capacitor between the two resistors, or switch the leads from the battery)?