Experiment 8 Solutions: RL and Undriven RLC Circuits

Part 1: Measure Resistance and Inductance Without a Core

Question 1:
What is the maximum current during the cycle? What is the EMF generated by the inductor at the time this current is reached?

The maximum current is about 180 mA. When the current is a maximum the inductor sees no changing current and hence isn’t doing anything (EMF is zero).

Question 2:
What is the time constant $\tau$ of the circuit?

Using the logarithmic method, $\tau = (-\text{slope})^{-1} = (-680 \text{ s}^{-1})^{-1} = 1.5 \text{ ms}$

Question 3:
What are the resistance $r$ and inductance $L$ of the coil?

Get the resistance from the answer to question 4: $r = \frac{V}{I} = \frac{(1.0 \text{ V})}{(180 \text{ mA})} = 5.6 \Omega$

Then we can get $L$ from the time constant: $L = \tau r = (1.5 \text{ ms})(5.6 \Omega) = 8.4 \text{ mH}$
Part 2: Measure Resistance and Inductance With a Core

Question 4:
Does the maximum current in the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to bigger or smaller makes sense)

No, the core doesn’t change the maximum current because it doesn’t change the resistance of the coil.

Question 5:
Does the time constant \( \tau \) of the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to longer or shorter makes sense)

Yes, the time constant does get longer (\( \tau = 7.0 \text{ ms} \)), because the core increases the inductance of the coil. It does this by increasing the magnetic field inside of the coil for a given current in the coil, which increases the magnetic flux and hence self inductance.

Question 6:
What are the new resistance \( r \) and inductance \( L \) of the coil?

\[ r \] doesn’t change: \( r = 5.6 \, \Omega \)
\[ L \] increases: \[ L = \tau r = (7.0 \, \text{ms})(5.6 \, \Omega) = 39. \, \text{mH} \]
Part 3: Free Oscillations in an Undriven RLC Circuit

In this part we turn on a battery long enough to charge the capacitor and then turn it off and watch the current oscillate and decay away.

1. Press the green “Go” button above the graph to perform this process.

Before you begin, for the circuit as given (with a 10 \( \mu \text{F} \) capacitor and a coil with resistance \( \sim 5 \Omega \) and inductance \( \sim 8.5 \text{ mH} \) as measured in Lab 8), what is the frequency at which the circuit should ring down?

An undriven RLC circuit will ring down at its natural frequency. Actually, when there is resistance in the circuit the natural frequency is modified slightly (we will ignore the effect of the resistance). So the ring down frequency is:

\[
\omega_0 \approx \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(8.5 \text{ mH})(10 \mu \text{F})}} \approx 3400 \text{ s}^{-1}
\]

\[
f_0 = \frac{\omega_0}{2\pi} \approx 550 \text{ Hz}
\]

**Question 7:**

What is the period of the oscillations (measure the time between distant zeroes of the current and divide by the number of periods between those zeroes)? What is the frequency?

It decays over several periods, so we find zeroes five periods apart. The time difference is 8.70 ms, and hence the period, is 1.74 ms, corresponding to a frequency

\[
f = \frac{1}{T} = \frac{5}{(8.70 \text{ ms})} = 575 \text{ Hz}
\]

**Question 8:**
Is this experimentally measured frequency the same as, larger than or smaller than what you calculated it should be? If it is not the same, why not?

The experimentally measured frequency is larger than predicted. This doesn’t make sense – we ignored the effects of the resistor, which tends to decrease the frequency. It must be that the inductance or capacitance are slightly different than we think.

**Part 4: Energy Ringdown in an Undriven RLC Circuit**

1. Repeat the process of part 1, this time recording the energy stored in the capacitor \( U_C = \frac{1}{2} CV^2 \) and inductor \( U_L = \frac{1}{2} LI^2 \), and the sum of the two.

**Question 9:**

The circuit is losing energy most rapidly at times when the slope of total energy is steepest. Is the electric (capacitor) or magnetic (inductor) energy a local maximum at those times? Briefly explain why.

The magnetic energy is a local maximum when the system is losing energy most rapidly. Energy loss comes from power dissipation in the resistor, which happens when current is large, which is when the magnetic energy (the yellow curve above) is dominant.