Purpose of the Experiment:

The direct goal of this experiment is to study a "conical pendulum", and apply Newton’s 2nd law to an object moving in a circular orbit. A round weight of mass \( m \) (the “mass”) is attached to a rotating shaft by a spring; you will adjust the angular velocity \( \omega \) of the shaft rotation, measure the radius \( r_m(\omega) \) of the circular motion of the mass, calculate the centripetal force from \( F = ma \) and use the results to find the force constant of the spring. To simplify the analysis of your results, assume that the mass of the spring can be neglected.

You will do the following things in this experiment:

- Measure the radius \( r_m(\omega) \) of the orbit of the mass \( m \) as a function of \( \omega \) for the shaft, and interpret your measurements with a model based on two things (i) the centripetal force is \( F_{\text{cent}} = m a_{\text{cent}} = m r_m(\omega) \omega^2 \), and (ii) the spring obeys Hookes’ law (spring force proportional to amount of stretch).

- This model predicts \( r_m(\omega) \to \infty \) at a critical angular frequency, \( \omega_c \), for the shaft rotation; you will observe the approach to this behavior and understand what causes it.

- You may find that your spring has an initial tension that must be overcome before the spring will begin to stretch.

Testing the Model:

The theoretical model that is derived by assuming the spring obeys Hooke’s law, \( F_{\text{spring}} = k[r_m(\omega) - r_0] \), and equating it to the centripetal force, \( F_{\text{cent}} = m r_m(\omega) \omega^2 \), is

\[
r_m(\omega) = \frac{r_0}{1 - m \omega^2/k} = \frac{r_0}{1 - (\omega/\omega_c)^2}
\]

The model contains two parameters, \( r_0 \) and \( \omega_c \). The best way to test this model and to extract values for the parameters is to plot a different combination of the variables. Let \( y \equiv \omega^2 \) and \( x \equiv 1/r \). If our model is correct, a plot of \( y \) versus \( x \) will be a straight line with a negative slope equal to \(-\omega_c^2 r_0\). The line will intersect the \( y \) axis at \( \omega_c^2 \) and the \( x \) axis at \( r_0 \).
1/r_0. Use your spreadsheet to create such a plot and use Trendline to determine the best fit values for \( \omega_c \) and \( r_0 \). Record these values on the Worksheet for the experiment.

**Spring Force Law**

If Eq. (1) fits the data within experimental error, that is a joint confirmation of \( F_{\text{cent}} = m r_m(\omega) \omega^2 \) and \( F_{\text{spring}} = k[r_m(\omega) - r_0] \). If you know \( m \) you can find the spring force constant from \( \omega_c \).

There is one more measurement you should make. Disconnect the wires from the apparatus. Remove the sliding panel on the side of the plastic box so that you can hold the shaft to keep it from rotating.

Stand the box on its side (where the wires were connected) and hold the counterweight and the shaft so that the spring and mass hang down vertically along the scale. Use the viewer to measure the value of \( r_m(0) = r_m(\omega = 0) \) when the mass is still and hanging vertically.

Enter your measured value for \( r_m(0) \) on the Worksheet for the experiment.

Now compare the value you measured for \( r_m(0) \) with the value \( r_0 \) obtained from your fit. You probably expect they should be the same, but in my experiment they were not. I found \( r_0 \) to be slightly smaller (about 2 mm) than \( r_m(0) \). Was that simply experimental error, or could it be real? Well, I think it might be real, and here is the explanation.

The parameter \( r_0 \) from the fit to Eq. (1) is the radius at which the spring force extrapolates to zero assuming the spring force is given by

\[
F_{\text{spring}} = k \Delta x,
\]

where \( \Delta x \) is the amount the spring has stretched. The amount the spring has stretched should be given by

\[
\Delta x = r_m(\omega) - r_m(0).
\]

You have measured \( r_m(0) \) as well as \( r_m(\omega) \) for several values of \( \omega \). However, suppose that the spring force is given by

\[
F_{\text{spring}} = F_0 + k \Delta x.
\]

That would mean there is an initial tension \( F_0 \) in the spring that must be overcome before the spring will begin to stretch. If there is an initial tension then we may write

\[
F_{\text{spring}} = k[r_m(\omega) - r_0] = F_0 + k[r_m(\omega) - r_m(0)],
\]

which means the parameter \( r_0 \) found in the fit is

\[
r_0 = r_m(0) - F_0/k.
\]
I found $\omega_c = 135 \text{r s}^{-1}$ and with $m = 0.0085 \text{kg}$ that means $k = 155 \text{N m}^{-1}$ and the initial tension is about $F_0 = k[r_m(0) - r_0] = 155 \times 0.002 \simeq 0.3 \text{N}$.

I looked in a spring manufacturer’s catalogue (http://www.centuryspring.com) and found that small springs like the ones in our experiments are typically wound with an initial tension of a few tenths of a Newton.

Compute the initial spring tension in your spring, if any, that would give rise to the difference between $r_0$ and $r_m(0)$ that you found. Enter this value on the Worksheet for the experiment.