Experiment 2: Uniform Circular Motion

Purpose of the Experiment:
The direct goal of this experiment is to study a “conical pendulum”, and apply Newton’s 2nd law to an object moving in a circular orbit. A round weight of mass \( m \) (the “mass”) is attached to a rotating shaft by a spring; you will adjust the angular velocity \( \omega \) of the shaft rotation, measure the radius \( r_m(\omega) \) of the circular motion of the mass, calculate the centripetal force from \( F = ma \) and use the results to find the force constant of the spring. To simplify the analysis of your results, assume that the mass of the spring can be neglected.

You will do the following things in this experiment:

- Measure the radius \( r_m(\omega) \) of the orbit of the mass \( m \) as a function of \( \omega \) for the shaft, and interpret your measurements with a model based on two things (i) the centripetal force is \( F_{\text{cent}} = ma_{\text{cent}} = mr_m(\omega) \omega^2 \), and (ii) the spring obeys Hookes’ law (spring force proportional to amount of stretch).

- This model predicts \( r_m(\omega) \to \infty \) at a critical angular frequency, \( \omega_c \), for the shaft rotation; you will observe the approach to this behavior and understand what causes it.

- You may find that your spring has an initial tension that must be overcome before the spring will begin to stretch.

Experimental Materials:

- Logger Lite Software
- Vernier LabPro Interface
- Circular Motion Apparatus/power cord
- Vernier Differential Voltage Probe
- small weight on spring
Setting Up the Experiment:

The apparatus includes a viewer to facilitate measuring the radius of the mass’s motion, \( r_m(\omega) \). It is a white teflon block with a black viewing tube and it slides in a slot in the top cover of the apparatus. A small nail protrudes down into the slot and should be placed to engage the loop of the brass wire that has a LED taped to it; that is so the LED will move to illuminate the mass as you try to read its position. (This all works better if the room lights are dim.) Another nail protrudes into the viewing tube and can be lined up over the black stripe around the mass; the same nail also allows you to read the radius on the scale.

You need to know the mass \( m \) of the round weight. The one I used had \( m = 8.5 \) gm; use this value for your experiment. (If you use an incorrect value for the mass, the experiment will still work normally, but the spring constant you find will be incorrect.)

Attach the mass and spring to the hook on the shaft inside the box. Then connect the apparatus to its 12V power supply via the connector on the side.

**You should replace the side cover to the box before you turn on the motor.** The weight moves at fairly high speed and would give a nasty whack to your finger if it were hit by it. Turn on the circular motion apparatus and try different speeds to see how it behaves. You will use the Logger Lite program `CircMotionLite.gmbl` to measure the rotation period, \( T \), of the mass, and from that calculate the angular rotation frequency \( \omega = 2\pi/T \).

A voltage probe should be connected from channel 1 of the Vernier LabPro interface to the two banana jacks on the apparatus that are farthest away from the speed control knob. Match the colors of the plugs and jacks.

As the motor rotates, a magnet in the counterweight triggers a reed relay on the top cover of the box and makes a voltage pulse every time the magnet passes under the relay. The voltage is normally close to 0, but rises to between 3 and 4 V when the magnet closes the reed relay. The program `CircMotionLite` acts like an oscilloscope to detect these voltage pulses. You will have to count the number of pulses in a known period of time and calculate the angular frequency of rotation, \( \omega \), of the motor shaft.

The way to make your measurements is to set the shaft speed with the knob on the apparatus, **wait at least 30 seconds for the speed to stabilize**, and measure the radius \( r_m(\omega) \) of the circular motion. After you have measured \( r_m(\omega) \), compute the angular velocity \( \omega \) of the rotation.

- You should make measurements for at least five different values of \( \omega \) that give \( r_m(\omega) \) between 5 and 10 cm.
- You should enter your \( \omega \) and \( r_m(\omega) \) data in an Excel spreadsheet and carry out a fit to see how well the model described in the Pre-Lab assignment describes the behavior you observed.
- You should analyze your results to determine the force constant of the spring in your apparatus and whether there is evidence for an initial tension in the spring (as described on page 5 of these instructions).
• You should turn the Worksheet for this experiment describing your results. The Worksheets will be available in the classroom. A copy is included as the last page of these instructions.

• Use the data input link on today’s web page to enter the value you obtained for $\omega_c$. There are several different springs on the apparatuses for this experiment. When your instructor plots a histogram of $\omega_c$ we can see how many different springs there are.

Making Measurements:
Start the program *CircMotionLite*. Go to the Experiment menu and select “Data Collection.” You may set parameters, including collection length and sampling rate, to your liking (default values are 1 sec and 1000 samples/sec; the maximum sample rate is 10,000 samples/sec).

The “Collect” option (green play button) will start the program measuring the voltage on channel 1, alternatively, you can hit the space bar to commence measuring.

The program will start to record the voltage immediately and will continue for the length of time set in the Data Collection window (1 sec or less is usually enough). You should then see a plot like the one in the graph below.

You could determine the shaft rotation period $T$ from this graph by finding the time interval between the first and last voltage peaks and then dividing by the number of peaks; the angular velocity $\omega$ will be $2\pi/T$. 
In a spreadsheet, record the angular velocity that you computed from the period and the radius you measured. Also, enter these values on the Worksheet for the experiment.

Do more measurements to find the radius $r_m(\omega)$ of the mass's circular motion for at least four more different angular velocities to give values of $r_m(\omega)$ between 5 and 10 cm. Allow the speed to stabilize for at least 30 s before you make each measurement.

Once you have completed these measurements of $r_m(\omega)$ vs. $\omega$, you should make a plot of $r_m(\omega)$ vs. $\omega$ on your spreadsheet. Does it exhibit the expected behavior?

**Testing the Model:**

The theoretical model that is derived by assuming the spring obeys Hooke's law, $F_{\text{spring}} = k[r_m(\omega) - r_0]$, and equating it to the centripetal force, $F_{\text{cent}} = m r_m(\omega) \omega^2$, is

$$r_m(\omega) = \frac{r_0}{1 - m \omega^2 / k} = \frac{r_0}{1 - (\omega / \omega_c)^2} \quad (1)$$

The model contains two parameters, $r_0$ and $\omega_c$. The best way to test this model and to extract values for the parameters is to plot a different combination of the variables. Let $y \equiv \omega^2$ and $x \equiv 1/r$. If our model is correct, a plot of $y$ versus $x$ will be a straight line with a negative slope equal to $-\omega_c^2 r_0$. The line will intersect the $y$ axis at $\omega_c^2$ and the $x$ axis at $1/r_0$. Use your spreadsheet to create such a plot and use Trendline to determine the best fit values for $\omega_c$ and $r_0$. Record these values on the Worksheet for the experiment.

**Spring Force Law**

If Eq. (1) fits the data within experimental error, that is a joint confirmation of $F_{\text{cent}} = m r_m(\omega) \omega^2$ and $F_{\text{spring}} = k[r_m(\omega) - r_0]$. If you know $m$ you can find the spring force constant from $\omega_c$.

There is one more measurement you should make. Disconnect the wires from the apparatus. Remove the sliding panel on the side of the plastic box so that you can hold the shaft to keep it from rotating.

Stand the box on its side (where the wires were connected) and hold the counterweight and the shaft so that the spring and mass hang down vertically along the scale. Use the viewer to measure the value of $r_m(0) = r_m(\omega = 0)$ when the mass is still and hanging vertically.

Enter your measured value for $r_m(0)$ on the Worksheet for the experiment.

Now compare the value you measured for $r_m(0)$ with the value $r_0$ obtained from your fit. You probably expect they should be the same, but in my experiment they were not. I found $r_0$ to be slightly smaller (about 2 mm) than $r_m(0)$. Was that simply experimental error, or could it be real? Well, I think it might be real, and here is the explanation.

The parameter $r_0$ from the fit to Eq. (1) is the radius at which the spring force extrapolates
to zero assuming the spring force is given by

\[ F_{\text{spring}} = k \Delta x, \]

where \( \Delta x \) is the amount the spring has stretched. The amount the spring has stretched should be given by

\[ \Delta x = r_m(\omega) - r_m(0). \]

You have measured \( r_m(0) \) as well as \( r_m(\omega) \) for several values of \( \omega \). However, suppose that the spring force is given by

\[ F_{\text{spring}} = F_0 + k \Delta x. \]

That would mean there is an initial tension \( F_0 \) in the spring that must be overcome before the spring will begin to stretch. If there is an initial tension then we may write

\[ F_{\text{spring}} = k[r_m(\omega) - r_0] = F_0 + k[r_m(\omega) - r_m(0)], \]

which means the parameter \( r_0 \) found in the fit is

\[ r_0 = r_m(0) - F_0/k. \]

I found \( \omega_c = 135 \text{ rad s}^{-1} \) and with \( m = 0.0085 \text{ kg} \) that means \( k = 155 \text{ N m}^{-1} \) and the initial tension is about \( F_0 = k[r_m(0) - r_0] = 155 \times 0.002 \simeq 0.3 \text{ N} \).

I looked in a spring manufacturer’s catalogue (http://www.centuryspring.com) and found that small springs like the ones in our experiments are typically wound with an initial tension of a few tenths of a Newton.

Compute the initial spring tension in your spring, if any, that would give rise to the difference between \( r_0 \) and \( r_m(0) \) that you found. Enter this value on the Worksheet for the experiment.
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Name ________________________________

Section __________ Table ____________

Data

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<th>Frequency (radians/sec)</th>
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Analysis

Critical angular velocity \( \omega_c \) from fit to data __________

Relaxed spring length \( r_0 \) from fit to data __________

Computed value of the spring constant \( k \) in Newtons/meter __________

Directly measured value of relaxed spring length \( r_m(0) \) __________

Initial spring tension in Newtons __________