Demonstration: Force and Impulse

This demo uses a force platform that PASCO introduced this summer. It is about 14 inches square and you can stand on it like a bathroom scale. It has the following characteristics.

- Force range: $-1000$ to $+4000$ N
- Output: $500$ N per V.
- Sensitivity: about $0.5$ N, reproducibility a bit worse.
- Response: up to $1000$ readings/sec (keep to $\leq 100$ for best sensitivity).

These notes will outline several interesting things you can do with the equipment. What inspired me to buy the force platform was a desire to have a demo for the classic falling chain problem below.

Falling Chain:

A chain of length $h$ and mass $m$ is held by one end at a distance $h$ above the floor. The chain is released.

What is the force exerted by the chain on the floor after the top end of the chain has fallen a distance $y$?

What is the force exerted by the chain on the floor at a time $t$ after the chain was released?

**Answer:** The force has two parts: (i) the weight of the chain at rest on the floor and (ii) the force to stop the moving links as they strike the floor.

After the top has fallen $y$, the weight of the chain on the floor is $mgy/h$ and the falling chain has speed $\sqrt{2gy}$.

In a small time $\delta t$ the mass of chain striking the floor is $(m \delta t/h)\sqrt{2gy}$ and the impulse to stop it (without bouncing) is $\delta P = \delta m \sqrt{2gy} = (2mgy/h) \delta t$.

Thus the force the chain exerts on the floor is $3mgy/h$ while the chain is still falling and $mg$ afterwards. It peaks at $3mg$.

Since $y = \frac{1}{2}gt^2$, the force as a function of time will be

$$F(t) = \begin{cases} \frac{3mgy^2}{2h} & 0 \leq t \leq \sqrt{\frac{2h}{g}} \\ mg & t > \sqrt{\frac{2h}{g}} \end{cases}$$

(1)
The force increases as $t^2$ to a peak value

$$F_{\text{peak}} = 3mg$$  

and the total impulse over the time the chain is falling is

$$P_{\text{total}} = \frac{3mg^2}{2h} \int_0^{\sqrt{2h/g}} t^2 \, dt = m \sqrt{2gh}$$  

This impulse includes the integral of the weight of the chain that is lying still on the floor. If you want only the integral of the force that stops the moving part of the chain, it will be $2/3$ of the value above.

**LabView Program**

There is a LabView program *FallingChain* that connects to the force platform through an SW-750 interface and can measure the force as a function of time. It can also carry out some analysis of $F(t)$. 

![LabView Graph](image-url)
The operation of the program is controlled mainly by the pull-down menu that you can see. Some comments are:

- The maximum sample rate is 100 Hz, not fast enough to study the bouncing of a ball that is inflated hard. An underinflated basketball might work.

- When you select Measure from the pull-down menu the RUN button glows bright green. When you click the button (or type the Esc key) the program starts to measure force at the sample rate chosen. It will continue measuring and plotting the force until you click the red STOP button (or type Esc again). Only data from the most recent 5 sec, or whatever you type in the Window(s) field, are kept in memory and plotted on the graph.

- The graph scales the y-axis automatically to fit the range of data on the plot. The meter full range is from zero to the multiple of 10 N that exceeds the maximum force on the plot.

The plot you see on the previous page is a heavy chain (it weighs about 28 N and is 1.88 m long) falling onto the platform. The first thing you will notice is that the plot is very noisy (and you can hear that, too, as the chain falls) hardly corresponding to the simple model we have. That’s the result, of course, of the chain links bouncing as they strike the platform or the chain already lying on it. This is a good demo for showing that real behavior does not always conform to simple models.

**Using the Program & Platform:**

If you click the Results tab you will see a window like this one. Whatever data are plotted on the graph are also entered in the table. Other features on the tab will be discussed on the next page.
Calibration:
The platform has a tare button that you should press when there is no load on it. In the lower right corner of the results tab are three fields where you may enter some parameters. The first is a standard deviation for force measurements (used by the fitting features). The second is the output voltage that corresponds to zero force (the platform can measure $-1000 \text{N} \leq F \leq +1000 \text{N}$). The third is the number of N corresponding to a 1 V change in output. For most purposes you can just use the default values.

Fitting:
The program can carry out several fits to the $F(t)$ data. Only the data between the two cursors are included in the fit. The Fit Function? pull-down menu lets you choose the fit you want.

- The Constant choice fits the data to a constant ($A$) and the second choice fits the data to a straight line, $A + B \cdot X$, where $X$ is the time.

- The $BG + A \cdot (X - X_0) \cdot (X - X_0)$ choice is intended for analyzing $F(t)$ while the chain is falling. The term $BG$ is a value you type into the field labeled Backgnd (N). It is the force before the chain starts to land on the platform. It should be 0, but sometimes the calibration drifts. You can obtain it by a Constant fit to the data before the chain starts to land. The quantity $X_0$ is the time you place the left cursor at. This fit choice is to Eq. (1).

- The third fit choice calculates the total impulse between the cursors, where the background in Backgnd (N) is subtracted from $F(t)$ before computing the integral.

- The fourth choice, Elevator $V(t)$, provides a running integral of the force divided by the mass (obtained from the background weight) to give a plot of velocity $v(t)$ between the cursors—assuming, of course, that the initial velocity is zero. The program only knows about the force exerted by the platform on the person standing on it. It works for the elevator but may give strange results in other situations.

- The fifth choice, Elevator $Y(t)$, provides a running integral of $v(t)$ to give a plot of position $y(t)$ between the cursors—assuming, of course, that the initial position is zero. It is useful for elevator measurements.

- Above the table are two small pull-down menus to control the Savitzky-Golay smoothing of the data (see the Appendix). The plot control menu allows you to choose whether to plot and fit the raw data or smoothed data.

It is tricky to decide where to place the cursors before doing the fit, because the $F(t)$ data are so noisy. However, we know the chain length ($h = 1.88\text{ m}$) fairly well and so we can calculate that it should take time $\sqrt{2h/g} = 0.62\text{s}$ for all of the chain to fall to the platform. The cursors should be placed $0.62\text{s}$ apart. It’s fairly hard to see when $F(t)$ first starts to rise, and easier to decide when the chain has come to rest on the platform. That’s how the cursors were placed as in the figure on page 2.
Here is the result of fitting the raw data from the graph on page 2 to Eq. (1).

And this is the impulse calculation for the same data. The impulse was found to be 16 Ns.
The chain weighs 28 N \((m = 2.86 \text{ kg})\), \(h = 1.88 \text{ m}\), the duration of the impulse \((\sqrt{2h/g})\) was 0.38 s, and the peak force from the fit \(A = 692 \text{ N s}^{-2}\) was 100 N. This is a bit larger than \(3 \times 28 \text{ N}\), probably because of the “noise”. The impulse calculated for these data was 12.8 N s.

The force measurement is so noisy because, as you can hear, the chain links bounce off one another. When a link bounces, the impulse is greater. Sometimes a link that has bounced pulls up one or more links below it and then the impulse is less than you would expect. The overall impulse is greater than the 12.8 N s for a chain that landed softly because some links that bounce up gain additional downward momentum from the force of gravity after they have bounced; that is why the measured impulse is 16 N s. All of this is complicated to deal with quantitatively and it is clear that the “structure” of the chain leads to a different \(F(t)\) than the model predicts. It seems to peak before all of the chain has come to rest on the platform. About the best you can do is to discuss whether the extra total impulse makes sense.

**Jumping Off**

One activity that can be studied quite well is someone jumping off the platform. Set the sample rate to 100 Hz and the Window to 10 s. Start measuring and have the subject stand still on the platform for a second or two, jump as high as they can and land on the floor beside the platform, then stop the measurement with the STOP button or the escape key. Here is a typical \(F(t)\) graph.

You can use the Constant fit to the portion of the graph where the person was standing still to find his weight; in this case it was 911 N. This value should be entered into the Background (N) field on the Results tab; this is necessary in order to avoid counting the time integral of the person’s weight as part of the impulse during the jump. You should discuss this with the students.

There are several features on this graph. First, the force exerted on the platform drops when the person bends his knees preparing to jump; it rises during the jump and falls to zero when the person is airborne.
Next you can set the cursors and the program will calculate the impulse during the jump.

Discuss with the students why this is the correct time interval to integrate $F(t) - BG$ over to get the total impulse. In this example, the integral was 199 N s. The program can make one other calculation that is useful. Developed for the elevator example (see below) it can calculate the impulse

$$P(t) = \int_0^t F(t') \, dt'$$

during the jump and divide the result by the person’s mass (obtained from the $BG$ weight) to get the velocity $v(t)$. If you select that from the fitting function, you will see a graph like this one.
The velocity found from the total impulse and from this curve at the end of the jump will, of course, be the same. You can use it to calculate how much the jumper raised his center of mass by at the peak of his jump. Here it was \( h = \frac{v^2}{2g} \simeq 0.23 \text{ m} \).

You could discuss how this compares with a basketball player who does a “slam dunk” and invite a few students to see how well they can do.

**Elevator Ride**

I took the apparatus on an elevator ride in the building 16 elevator, which is one of the newer and faster elevators at MIT. (The SW750 was powered by lawnmower battery.) Here is the force exerted by my feet on the platform as I rode from the 5th floor to the ground floor.

Here is the total impulse for the first part of elevator ride; it was about \(-172 \text{ Ns}\).
You can see there is about 3.5 s at constant velocity before the elevator begins to slow down. I asked the program to integrate $F(t)$ to find the velocity during the entire elevator ride.

You can see the “maximum” velocity was about $-2.5 \text{ m s}^{-1}$ and the program calculated that I was still moving upward at about $20 \text{ cm s}^{-1}$ at the end of the ride. It’s not easy to build an accurate inertial guidance system! (The zero of the force platform drifts a bit.) Charging ahead with inertial guidance, the last choice on the fitting menu will integrate $F(t)$ twice to calculate the displacement. Here is the result.

It seems each floor in building 16 is about 3.75 m or 12.5 ft. This seems reasonable, perhaps a little small.
Data Smoothing

A method to obtain derivatives of data that have some noise, and to reduce some of the noise itself, was introduced by two scientists at the Perkin Elmer Company in the 1960s [A. Savitzky and M. J. E. Golay, Analytical Chemistry 36, 1627-1639 (1964)]. The method is to carry out a local least squares fit of a polynomial to just a few data points near the time you are interested in, and then find the local derivatives from the fit coefficients. This LabVIEW program does that in order to make the velocity and acceleration plots. Even with this approach, the acceleration graphs are rather noisy.

Using the Savitzky-Golay analysis, one can make two choices: (i) how many data points to include in the local fit, and (ii) what degree of polynomial to use in the fit. The defaults used in the program are 7 points (i.e., the point where you want the derivative and 3 points before and after it) and a quadratic polynomial—which can give first and second derivatives.

The pull-down menus at the top of the Results tab (figure at left) offer other choices for anyone who is inclined to experiment.

The coefficient of the constant term in the local fit represents a local average and provides smoothing of the original data. Normally the “raw” position data are plotted on the graph and used in a fit, but a pull-down menu allows plotting the smoothed data instead.

You may want to experiment with this data manipulation; if you make any changes in the Savitzky-Golay pull-down menus, or even just the standard deviation for the position measurements, choose the bottom item on the main menu (Recalculate V, A) in order to apply them. Incidentally, whether you fit the raw or smoothed position data, you should get the same results.